ISSN 2277-2685
IJESR/June. 2025/ Vol-15/Issue-3s/623-629
Aare Sreeja *et. al.,* / International Journal of Engineering & Science Research

Advances in Distributed graph Filtering

Ms. S Surekha, Aare Sreeja, Bommanamaina Sowmya, Palbettu Sony
[1]Assistant Professor, Department Of ECE, Bhoj Reddy Engineering College For Women, India.
[2,3,4]B. Tech Students, Department Of ECE, Bhoj Reddy Engineering College For Women, India.

ABSTRACT

Graph filters are one of the core tools in graph signal processing. A central aspect of them is their direct distributed implementation. However, the filtering performance is often traded with distributed communication and computational savings. To improve this tradeoff, this work generalizes state-of-the art distributed graph filters to filters where every node weights the signal of its neighbors with different values while keeping the aggregation operation linear. This new implementation, labeled as edge-variant graph filter, yields a significant reduction in terms of communication rounds while preserving the approximation accuracy. In addition, we characterize the subset of shift-invariant graph filters that can be described with edge-variant recursions. By using a low-dimensional parametrization the proposed graph filters provide insights in approximating linear operators through the succession and composition of local operators, i.e., fixed support matrices, which span applications beyond the field of graph signal processing. A set of numerical results shows the benefits of the edge-variant filters over current methods and illustrates their potential to a wider range of applications than graph filtering.

1-INTRODUCTION

Forgetting the philosophical standoff, we can assert that actions require interactions. Therefore, it is naive to believe that it is possible to understand the inner workings of processes observed in our daily life, e.g., currency trading, friendship formation, oil pricing, without the understanding of the structure that defines (or supports) the interactions in such systems. For example, it is not possible to fully understand conflict without a proper assessment of how the relationships among the involved parties.

Similarly, we cannot expect to produce high-quality predictions of users' consumption patterns, if we do not make use of the information available from users with the same characteristics, e.g., close friends, similar demographics, etc.

Although traditional signal processing has always made use of models and relations in data, e.g., the correlation between measurements, traditional tools are not sufficient to address the challenges that complex interactions, beyond time and space, bring into the table. As an answer, graph signal processing (GSP) has established itself as a balanced mix of the well-known mathematical rigor from signal processing and graph theory, with empirical modeling seen in network theory. This blend has led to a powerful tool for analyzing data from network processes exploiting all available information about the existing interactions.

This thesis, using GSP as its foundation, aims to provide a further understanding of processes where the interactions between elements of a system are at their core. It presents advanced topics in areas related to how network data has to be processed, how we can implement these data processing pipelines, and how to discover relations between actors in a network process by observation of the network data itself.

In this chapter, we first motivate the use and study of graphs, as well as its combination with signal processing, for analyzing network data. We then provide the scope of the research in the field of GSP and the outline of this thesis. We conclude with our research contributions within GSP and other areas intersecting with signal processing. We do all this considering that the contributions of this thesis must reach an audience not versed in the area.

2-LITERATURE SURVEY

Distributed graph filtering has gained significant attention due to the growing need for scalable graph processing in large-scale applications. Traditional graph filtering techniques, such as spectral filtering and wavelet-based methods, are computationally expensive and struggle with distributed implementations. Graph Neural Networks (GNNs), including Graph SAGE, GAT, and GCN, have introduced efficient filtering mechanisms but face challenges in scalability and real- time processing. Polynomial approximations, such as Chebyshev and Lanczos methods, have been explored to reduce computational complexity in spectral filtering. Recent advancements in federated learning and differential privacy enable secure and decentralized graph filtering, addressing privacy concerns in distributed environments. Streaming graph filtering techniques have been developed to handle dynamic and evolving graphs efficiently. Partitioning strategies, like METIS and random-walk-based methods, optimize workload distribution for parallel processing.

Frameworks like Apache Spark GraphX, Deep Graph Library (DGL), and PyTorch Geometric provide scalable solutions but still face challenges in communication overhead. Adversarial robustness has become a key focus to prevent malicious attacks on graph-structured data. Despite these advancements, there remains a need for optimized, real-time, and privacy-preserving distributed graph filtering methods for large-scale applications.

3-GRAPH SIGNAL PROCESSING

Traditionally, time and space have been two physical domains that have allowed for defining a natural way to organize and explore data. For example, a series of stock prices during a year are considered as a time series; that is, a sequence of quantities that are ordered based on their recording time. Similarly, the concentration of a certain gas or the temperature in a geographical region are typical examples of field measurements. Here, the data is structured based on the (geographic) location where the measurements are taken. Naturally, the combination of such domains can be considered, i.e., spatio- temporal domain, thus a structure is naturally imposed on this data when it is examined over a window of time in a particular spatial region. Figure 2.1 provides examples of both temporal and spatial signals.

Graph Signal Processing

We consider a network to be represented by a graph $G = (V, E)$, where $V = \{v_1, \ldots, vN\}$ is the set of N vertices (nodes) and $E \subseteq V \times V$ is the edge set of M tuples $ei, j = (vi , vj-)$ repre- senting the connections between the nodes in the network. As a network, the connections in a graph can have direction; that is, a bidirectional exchange of information between nodes i and j does not necessarily have to exist. This behaviour is captured, and repre- sented abstractly, through either directed or undirected graphs. A graph is said to be undi- rected if there is no orientation of the edges (information flow) for all tuples $(vi , vj) \in E$, otherwise the graph is called directed. Figure 2.2 illustrates the difference between these two types of graphs. When there is information about the strength of the connection, i.e., wi, j ∈ **R**+ is the weight of the edge (vj , vi) ∈ E , we say that the graph is a weighted graph, otherwise we consider it as an unweighted graph.

Advances in Graph Filtering

Graph filters are one of the core tools in graph signal processing. A central aspect of them is their direct distributed implementation. However, the filtering performance is often traded with distributed communication and computational savings. To improve this tradeoff, this chapter provides a generalization of state-of-the-art distributed graph filters to filters where every node weights the signal of its neighbors with different values while keeping the aggregation operation linear. This new implementation, labeled as edge-variant graph filter, yields a significant reduction in terms of communication rounds while preserving the approximation accuracy. In addition, we characterize a subset of shift-invariant graph filters that can be described with edge- variant recursions. By using a low-dimensional parametrization, these shift-invariant filters provide new insights in approximating linear graph spectral operators through the succession and composition of local operators, i.e., fixed support matrices.

Filtering is one of the core operations in signal processing. The necessity to process large amounts of data defined over non-traditional domains, characterized by a graph, triggers advanced signal processing of the complex data relations embedded in that graph. Examples of the latter include biological, social, and transportation network data. The field of graph signal processing (GSP) has been established to incorporate the underlying structure in the processing techniques. Through a formal definition of the graph Fourier transform (GFT), harmonic analysis tools employed for filtering in traditional signal processing have been adapted to deal with signals defined over graphs .

Similarly to time-domain filtering, graph filters manipulate the signal by selectively amplify-ing/attenuating its graph Fourier coefficients. Graph filters have seen use in applications such as signal analysis , classification , reconstruction , denoising and clustering . Furthermore, they are the central block in graph filter banks , wavelets , and convolutional neural networks .

Distributed implementations of graph filters emerged as a way to deal with the ubiquity of big data applications and to improve the scalability of computation. By allowing nodes to exchange only local information, finite impulse response (FIR)and in- finite impulse response (IIR) architectures have been devised to implement a variety of responses. However, being inspired by time-domain filters, the above implementations do not fully exploit the structure in the graph data. The successive signal aggregations are locally weighted with similar weights. This procedure often leads to high orders when approximating the desired response. To overcome this challenge, this chapter proposes a generalization of the distributed graph filtering concept by applying edge-based weights to the information coming from different neighbors. While the de- tailed contributions are provided in Section 3.1.2, let us here highlight that the above twist yields graph filters that are flexible enough to capture complex responses with much lower complexity.

4-RESULTS

We now present a set of numerical examples to corroborate the applicability of the proposed filters for several distributed tasks. Table I presents a

ISSN 2277-2685
IJESR/June. 2025/ Vol-15/Issue-3s/623-629
Aare Sreeja *et. al.,* / International Journal of Engineering & Science Research

summary of the different graph filters mentioned in this work along with their specifications. In our simulations2 , we made use of the GSP toolbox. A. Graph Filter Approximation We here test the proposed FIR graph filters in approximating a user provided frequency response. We consider a random community graph of N = 256 nodes and shift operator S = L. The frequency responses of interest are two commonly used responses in the GSP community, i.e., (i) the exponential kernel h˜(λ) := e −γ(λ−µ) 2 , with γ and µ being the spectrum decaying factor and the central parameter, respectively; (ii) the ideal low-pass filter h˜(λ) = (1 0 ≤ λ ≤ λc 0 otherwise, with λc being the cut-off frequency. The approximation accuracy of the different filters is evaluated in terms of the normalized squared error NSE = kH˜ − Hfitk 2 F /kH˜ k 2 F . Hfit stands for the filter matrix of the fitted filters. Fig. 2 illustrates the performances of the different filters. In the exponential kernel scenario, we observe that the CEV FIR filter outperforms the other alternatives by showing a performance improvement of several orders of magnitude. A similar result is also seen in the low-pass example, where the CEV FIR filter achieves the error floor for K = 8, while the NV graph filter for K = 13 and the classical FIR filter for K = 17. Additionally, we observe that the SIEV FIR filter achieves a similar performance as the NV FIR filter. This result suggests that despite the additional DoF of the SIEV FIR filter, the nonconvex design strategy yields a local minimum that does not exploit the full capabilities of the filter. This local minimality effect can be seen in the stagnation of the error of the SIEV FIR filter for the exponential kernel case after K ≥ 8. Finally, we notice that the SICEV filter achieves a performance similar to the NV Filter of the same order, while having less DoF. This characteristic of the SICEV shows its benefits as the order increases. Having to estimate less parameters, the error stagnation for the step response is achieved at a higher filter order, hence a better approximation can be obtained.

The above observations further motivate the use of the CEV FIR filter, which trades better the simplicity of the design and the available DoF. In fact, even though the CEV FIR filter is conceptually simpler than the SIEV graph filter, it performs better than the latter. In addition, the larger DoF of the CEV FIR filter compared to the NV FIR filter (i.e., nnz(S)·K+N vs N ·(K + 1)) allows the CEV FIR filter to better approximate the desired response. In a distributed setting, these benefits translate into communication and computational savings.

Several distributed tasks of interest consist of performing a linear operation H˜ ∈ R N×N over a network. This can be for instance a beamforming matrix over a distributed array or a consensus matrix. In most of these cases, such linear operators cannot be straightforwardly distributed. In this section, we illustrate the capabilities of the developed graph filters in addressing this task.

Distributed Beamforming. We here consider the task of applying a beamforming matrix WH to
signals acquired via a distributed array

Phase 1 outputs:

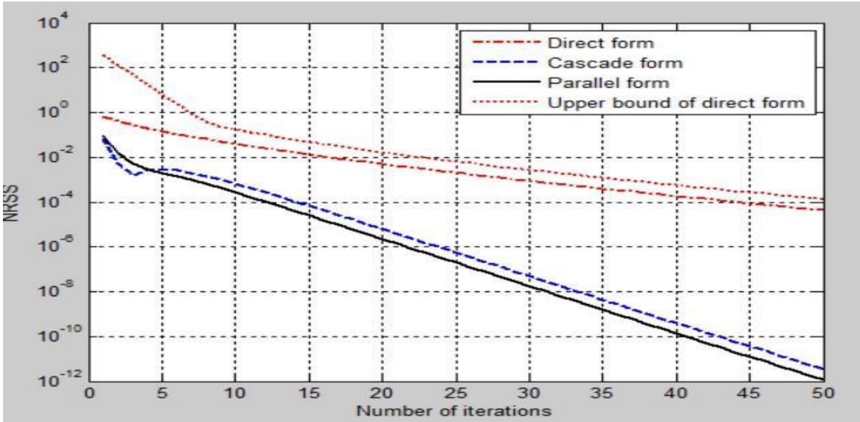

Figure2.7 NSE versus filter order for different distributed FIR filter

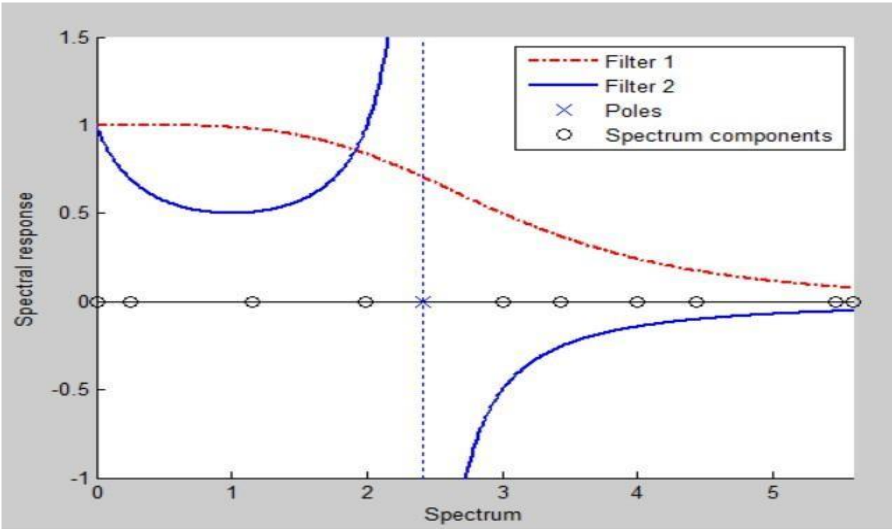

Figure 2.8Convergence error versus the number of iterations for the Tikhonov denoising problem.

Phase 2 output:

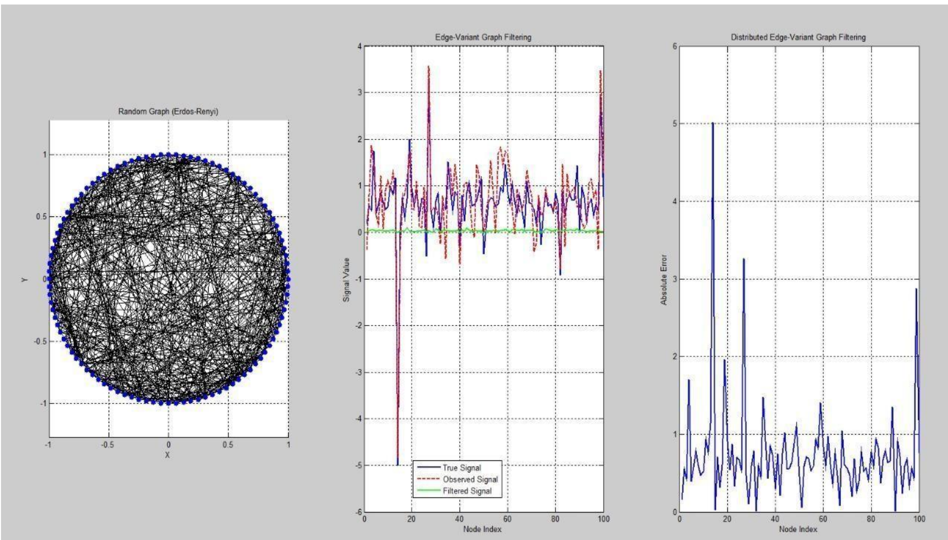

Figure2.9 Eigenvalues [λ]

The preferred to improve the convergence speed. However, values below 0.7 should in general be avoided since this restricts too much the feasible set of hence leading to a worse approximation error. Second, values of $\delta \approx 0.7$ seem to give the best tradeoff, since the convergence speed is doubled w.r.t the ARMA1 and the approximation error is close to machine precision. Finally, we did not plot the classical FIR filter for solving this problem, since its performance is identical to the ARMA1 for the
same distributed cost.

5-CONCLUSION

In this work, a generalization of distributed graph filters was proposed. These filters, that we referred to as edge-variant graph filters, have the ability to assign different weights to the information coming from different neighbors. Through the design of edge-weighting matrices, we have shown that it is possible to weight, possibly in an asymmetric fashion, the information propagated in the network and improve the performance of state-of-the-art graph filters. By introducing the notion of filter modal response, we showed that a subclass of the edge-variant graph filters have a graph Fourier interpretation that illustrates the filter action on the graph modes. Despite that the most general edgevariant graph filter encounters numerical challenges in the design phase, a constrained version of it was introduced to tackle this issue. This so-called constrained edge-variant graph filter enjoys a similar distributed implementation, generalizes the state-of-the-art approaches, and is characterized by a simple least-squares design. For the constrained version, we also showed that there exists a subclass which has a modal response interpretation.

Finally, we extended the edge-variant idea to the family of IIR graph filters, particularly to the ARMA1 graph filter. We showed that by adopting the same local structure a distributed rational filter can be achieved, yet with a much faster convergence speed. Several numerical tests corroborate our findings and show the potential of the proposed filters to improve state-of-the-art techniques. Future research in this direction should concern the following points: i) improve the design strategy of the more general edge-variant version; ii) improve the saturation accuracy of the proposed methods when dealing with a distributed implementation of linear operators; iii) conciliate the world of GSP with that of distributed optimization and exploit the latter to design distributed graph filters; and iv) extend the edgevariant concept beyond the ARMA1 implementation to the global family of IIR graph filters.

By introducing the notion of filter modal response, we showed that a subclass of the edge-variant graph filters have a graph Fourier interpretation that illustrates the filter action on the graph modes. Despite that the most general edge-variant graph filter encounters numerical challenges in the design phase, a constrained version of it was introduced to tackle this issue

REFERENCES

[1]M. Coutino, E. Isufi, and G. Leus, “Distributed edgevariant graph filters,” in IEEE 7th Int.
Workshop Comp. Adv. in Multi-Sensor Adap. Proc.(CAMSAP). IEEE, 2017.

[2]G. Taubin, “Geometric signal processing on polygonal meshes,” in EUROGRAPHICS, 2000.

[3] D. I. Shuman, S. K. Narang, P. Frossard, A. Ortega, and P. Vandergheynst, “The emerging

field of signal processing on graphs: Extending high-dimensional data analysis to networks and other irregular domains," IEEE Sig. Proc. Mag., vol. 30, no. 3, pp. 83–98, 2013.

[4] A. Sandryhaila and J. M. Moura, "Discrete signal processing on graphs," IEEE Trans. Signal Process, vol. 61, no. 7, pp. 1644–1656, 2013.

[5] G. Taubin, T. Zhang, and G. Golub, "Optimal surface smoothing as filter design," in European Conf. on Computer Vision. Springer, 1996, pp. 283–292.

[6] D. I. Shuman, P. Vandergheynst, and P. Frossard, "Distributed signal processing via chebyshev polynomial approximation," arXiv preprint arXiv:1111.5239, 2011.

[7] S. K. Narang, A. Gadde, and A. Ortega, "Signal processing techniques for interpolation in graph structured data," in Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP). IEEE, 2013, pp. 5445–5449.

[8] M. Onuki, S. Ono, M. Yamagishi, and Y. Tanaka, "Graph signal denoising via trilateral filter on graph spectral domain," IEEE Trans. on Sig. and Inf. Proc. over Netw., vol. 2, no. 2, pp. 137–148, 2016.

[9] S. Segarra, A. Marques, and A. Ribeiro, "Optimal graphfilter design and applications to distributed linear network operators," IEEE Trans. Signal Process, 2017.

[10] A. Loukas, A. Simonetto, and G. Leus, "Distributed autoregressive moving average graph filters," IEEE Sig. Proc. Lett., vol. 22, no. 11, pp. 1931–1935, 2015.