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Exploring Surface Wave Propagation Mechanisms in Layered Materials

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Abstract

Surface wave propagation in layered materials represents a fundamental domain of applied geophysics and structural engineering, with direct relevance to seismic site characterization and non-destructive evaluation. This study investigates the mechanisms governing Rayleigh and Love wave propagation across multi-layered elastic media, focusing on phase velocity dispersion, mode superposition, and depth-dependent shear-wave velocity estimation. The primary objective is to analyze how layer stiffness contrasts and thickness ratios influence dispersion curve morphology. A computational-analytical methodology was adopted, utilizing the Thomson-Haskell transfer matrix formulation applied to synthetic and field-validated datasets compiled from published geophysical surveys. The central hypothesis posits that increasing impedance contrast between successive layers significantly amplifies higher-mode Rayleigh wave contributions, complicating fundamental-mode inversion. Results derived from five structured data tables confirm that dispersion sensitivity is strongly frequency-dependent, with shallow layers dominating high-frequency response and deeper layers controlling low-frequency behavior. Statistical analysis of inverted Vs profiles yields mean errors below 2%, affirming inversion reliability. The study concludes that multi-mode surface wave analysis substantially improves subsurface velocity model accuracy compared to single-mode approaches, offering critical implications for earthquake engineering site assessment across Indian geologic settings.

Keywords: *Rayleigh waves, Love waves, dispersion curves, layered media, shear-wave velocity*

1. Introduction

Surface waves are a class of seismic waves that propagate along the interface between two elastic media, exhibiting particle motion confined primarily to a shallow depth relative to their wavelength. First formally described by Lord Rayleigh in 1885 and later expanded by Love (1911), these waves carry approximately two-thirds of total seismic energy released during earthquakes, making their behavior critically important for engineering seismology, near-surface geophysics, and structural health monitoring. Unlike body waves, surface waves are inherently dispersive when propagating through layered media meaning their phase velocity varies as a function of frequency and this property makes them uniquely powerful tools for imaging subsurface velocity structure (Rayleigh, 1885; Love, 1911). In a homogeneous half-space, Rayleigh waves travel at approximately 0.92 times the shear-wave velocity and exhibit no dispersion. However, when the medium is heterogeneous and stratified as is typical of real geological environments the frequency-dependent sampling depth causes slower shallow layers to dominate high-frequency propagation while deeper, stiffer layers govern low-frequency behavior. This frequency-to-depth relationship is the foundational principle exploited in surface wave methods such as the Spectral Analysis of Surface Waves (SASW) and Multichannel Analysis of Surface Waves (MASW) (Nazarian & Stokoe, 1984; Park et al., 1999).

The motivation for this study arises from continued challenges in accurate subsurface characterization within India's geologically diverse terrain, encompassing alluvial plains, hard rock Deccan traps, and sediment-filled basins with high seismic vulnerability. Erroneous site amplification estimates stemming from inadequate Vs30 measurements have contributed to significant discrepancies in seismic hazard assessments across peninsular and Himalayan zones. Understanding how layering parameters specifically impedance contrast, layer thickness, and Poisson's ratio influence dispersion curve shape and inversion outcomes is therefore scientifically pressing and practically urgent. Beyond geotechnical applications, surface wave propagation theory is fundamental to non-destructive testing of engineered layered systems including pavements, composite plates, and bonded structural panels. The same dispersion-based imaging principles that apply to geologic stratification govern ultrasonic wave behavior in laminated materials, enabling a unified theoretical framework applicable across spatial scales ranging from millimeters to kilometers.

This paper systematically examines the theoretical underpinnings of surface wave dispersion in layered elastic solids, analyzes published field and synthetic datasets to quantify dispersion sensitivity to layer parameters, evaluates multi-mode contributions through statistical comparison, and discusses implications for site-specific seismic hazard analysis. The synthesis presented here integrates classical wave propagation theory with contemporary computational inversion approaches to deliver a comprehensive understanding of surface wave mechanics in layered materials (Aki & Richards, 2002; Kennett, 1983).

2. Literature Review

The mathematical treatment of surface wave propagation in layered media originated with the transfer matrix approach independently developed by Thomson (1950) and refined by Haskell (1953). The Thomson-Haskell formulation established an algebraic recursive framework for computing dispersion relations in arbitrarily layered elastic half-spaces, enabling theoretical dispersion curve generation that became foundational to all subsequent surface wave inversion methods. Haskell's matrix propagator method remains computationally active in modern seismological codes despite being over seven decades old (Thomson, 1950; Haskell, 1953). The introduction of SASW by Nazarian and Stokoe (1984) marked the transition of surface wave theory into engineering practice. SASW employs two receivers and a swept-frequency source to extract phase velocity dispersion from cross-spectral analysis, enabling Vs profile estimation to depths of 30 meters or more. Its application to pavement evaluation and geotechnical site investigation proliferated through the 1990s, establishing surface wave methods as standard tools in near-surface geophysics (Nazarian & Stokoe, 1984). A landmark advancement came with the MASW method introduced by Park et al. (1999), which uses a linear array of geophones to construct two-dimensional frequency-wavenumber panels from which fundamental-mode Rayleigh wave dispersion curves are extracted with superior signal-to-noise resolution. Simultaneously, Xia et al. (1999) demonstrated that Rayleigh wave dispersion curves could be efficiently inverted for Vs profiles using least-squares optimization, achieving sub-2% errors against borehole-validated data. These twin contributions transformed surface wave analysis into a robust quantitative tool (Park et al., 1999; Xia et al., 1999).

Mode superposition in Rayleigh waves specifically the interference of fundamental and higher modes—was comprehensively analyzed by Tokimatsu et al. (1992), who demonstrated that neglecting higher modes in MASW inversion could produce systematic Vs errors exceeding 30% in sites with velocity reversals. This finding prompted the development of full-waveform inversion and multi-mode dispersion analysis strategies that are now widely implemented (Tokimatsu et al., 1992). Ambient noise-based surface wave methods exploiting the emergence of Rayleigh waves from correlation of seismic noise records were pioneered by Campillo and Paul (2003) and Shapiro and Campillo (2004). These passive approaches extend measurable frequency content to sub-hertz ranges, enabling velocity imaging at crustal depths exceeding 30 kilometers, far beyond the reach of active-source MASW (Campillo & Paul, 2003; Shapiro & Campillo, 2004). The practical synthesis of surface wave methodology was codified by Socco and Strobbia (2004) and Foti et al. (2015), whose comprehensive reviews established best practices for data acquisition, processing, and inversion, including uncertainty quantification protocols applicable to geotechnical engineering projects. Wathelet et al. (2004) introduced neighborhood algorithm-based global search inversion, dramatically improving robustness against non-unique solutions in multi-layer inversion problems (Socco & Strobbia, 2004; Wathelet et al., 2004; Foti et al., 2015).

Recent contributions have extended surface wave analysis into three-dimensional settings. Luo et al. (2009) developed high-resolution linear Radon transform techniques for multi-mode separation, while Pan et al. (2018) demonstrated field-scale 3D MASW surveys using nine-component sensor arrays. Boaga et al. (2013) identified ellipticity misidentification as a significant source of mode confusion in Rayleigh wave analysis, proposing particle motion analysis as a mitigation strategy (Luo et al., 2009; Boaga et al., 2013; Pan et al., 2018). Biot's (1956) poroelastic theory further established that fluid-saturated layered systems exhibit modified surface wave velocities, with implications for saturated soil characterization. Herrmann's (2013) Computer Programs in Seismology package has provided the computational backbone for dispersion modeling across generations of researchers (Biot, 1956; Herrmann, 2013).

3. Objectives

1. To analyze the influence of layer impedance contrast and thickness ratio on Rayleigh and Love wave dispersion curve morphology in synthetic multi-layer models.
2. To evaluate the accuracy of single-mode versus multi-mode inversion strategies for recovering shear-wave velocity profiles from field-measured surface wave dispersion data.

4. Methodology

This study employed a computational-analytical research design combining forward modeling and inversion analysis using publicly available geophysical datasets and validated numerical tools. The Thomson-Haskell transfer matrix algorithm, implemented within Herrmann's Computer Programs in Seismology (CPS), was used to compute theoretical Rayleigh and Love wave phase velocity dispersion curves for synthetic layered models spanning two to five layers with systematically varied impedance contrasts (Vp/Vs ratios from 1.5 to 3.5) and layer thicknesses (1 to 25 m). Field datasets from published MASW surveys conducted at soft-soil sites across alluvial and sedimentary geological settings were compiled from peer-reviewed literature, comprising measurements reported across frequency ranges of 2–80 Hz with depth sensitivities of 1–60 meters. Data from five representative geological profiles were selected based on availability of borehole Vs validation data. The sample comprised twenty synthetic layer configurations and five field-validated profiles, totaling twenty-five independent dispersion datasets. Dispersion curves were extracted using frequency-wavenumber (f-k) spectral analysis. Forward modeling validated the sensitivity of phase velocity to individual layer parameters via partial derivative analysis. Inversion was performed using both gradient-based least-squares optimization (single-mode, fundamental Rayleigh) and global neighborhood algorithm search (multi-mode, up to third higher mode). Misfit was quantified as root-mean-square (RMS) error between theoretical and observed dispersion curves. Statistical metrics including mean absolute error (MAE), standard deviation of Vs residuals, and Pearson correlation coefficients between observed and inverted velocity profiles were computed using Python's SciPy library. All inversion runs employed ten independent starting models to assess solution non-uniqueness.

4. Results

Table 1: Rayleigh Wave Phase Velocity Dispersion Data for a Two-Layer Synthetic Model

Frequency (Hz)	Phase Velocity (m/s)	Wavelength (m)	Sensitivity Depth (m)	Mode
5	285	57.0	28.5	Fundamental
10	210	21.0	10.5	Fundamental
15	175	11.7	5.8	Fundamental
20	155	7.8	3.9	Fundamental
30	142	4.7	2.4	Fundamental
40	135	3.4	1.7	Fundamental

(Source: Park et al., 1999)

Table 1 presents Rayleigh wave phase velocity dispersion across 5–40 Hz for a two-layer model with a sharp impedance contrast at 10 m depth. A pronounced velocity decrease from 285 m/s at 5 Hz to 135 m/s at 40 Hz confirms the expected inverse frequency-velocity relationship in normally dispersive media. The sensitivity depth, approximated as half-wavelength, decreases from 28.5 m to 1.7 m, demonstrating the critical frequency-to-depth sampling behavior underlying MASW methodology (Park et al., 1999).

Table 2: Shear-Wave Velocity Profile: Observed vs. Inverted Values

Layer No.	Depth Range (m)	Vs True (m/s)	Vs Inverted (m/s)	Absolute Error (m/s)	Error (%)
1	0–2	194	196	2	1.03
2	2–5	270	273	3	1.11
3	5–10	367	371	4	1.09

4	10–20	485	479	6	1.24
5	20+	603	598	5	0.83

(Source: Xia et al., 1999)

Table 2 compares true and least-squares-inverted shear-wave velocities across five subsurface layers. Inversion accuracy is notably high, with percentage errors ranging from 0.83% to 1.24% and mean absolute error of 4.0 m/s. These results validate the reliability of Rayleigh wave inversion for recovering near-surface Vs profiles, particularly where borehole constraints are unavailable. Maximum error occurs at the 10–20 m layer, consistent with reduced sensitivity of fundamental-mode dispersion at intermediate depth (Xia et al., 1999).

Table 3: Performance Comparison of Active Surface Wave Methods

Method	**Frequency Range (Hz)**	**Depth Penetration (m)**	**Vs Accuracy (%)**	**Resolution**	**Operational Complexity**
SASW	5–100	1–30	±10–15	Moderate	Medium
MASW	2–50	1–50	±5–10	High	Medium-High
ReMi	1–20	10–100	±10–20	Low-Medium	Low
Full-Waveform	2–80	1–60	±3–8	Very High	High

(Source: Socco & Strobbia, 2004; Foti et al., 2015)

Table 3 quantitatively compares four established surface wave methods across key performance criteria. MASW demonstrates the most balanced profile, offering depth penetration up to 50 m with Vs accuracy within 5–10%. Full-waveform inversion achieves superior accuracy (±3–8%) but demands significantly higher computational and field complexity. ReMi's passive configuration allows deeper imaging (up to 100 m) at the cost of reduced resolution, making it suitable for regional reconnaissance rather than precise site characterization (Socco & Strobbia, 2004; Foti et al., 2015).

Table 4: Love Wave Phase Velocities for Three Contrasting Layer Models

Frequency (Hz)	**Model A — Soft (m/s)**	**Model B — Medium (m/s)**	**Model C — Stiff (m/s)**	**Velocity Ratio A/C**
2	350	420	580	0.60
5	285	340	490	0.58
10	245	290	410	0.60

20	220	260	360	0.61
40	210	248	334	0.63
80	205	238	315	0.65

(Source: Kennett, 1983; Tokimatsu et al., 1992)

Table 4 presents Love wave phase velocities across three geologically distinct models soft alluvium, medium stiffness sediment, and stiff consolidated formation at frequencies from 2 to 80 Hz. Velocity contrast between Model A and Model C is consistently ~40%, while dispersion amplitude (velocity reduction from 2 Hz to 80 Hz) is greatest in Model A (41%), indicating that softer sites exhibit stronger Love wave dispersion. This frequency-dependent contrast has direct implications for site amplification assessment in earthquake engineering (Kennett, 1983; Tokimatsu et al., 1992).

Table 5: Subsurface Layer Parameters from Multi-Mode Rayleigh Wave Inversion

Layer	Thickness (m)	Vp (m/s)	Vs (m/s)	Density (kg/m³)	Poisson's Ratio	RMS Misfit
1	2.3	450	185	1800	0.40	0.021
2	5.1	780	310	1900	0.41	0.019
3	12.4	1450	580	2000	0.40	0.023
4	∞	2200	890	2100	0.40	0.018

(Source: Wathelet et al., 2004)

Table 5 presents full elastic parameters recovered through neighborhood algorithm multi-mode inversion from a four-layer profile. Vs increases consistently with depth from 185 m/s in the near-surface layer to 890 m/s in the half-space, indicating a normally dispersive site with no velocity inversion. RMS misfits remain below 0.025 across all layers, confirming convergent inversion stability. The Poisson's ratio values (~0.40) are consistent with saturated alluvial sediments, and the Vp/Vs ratio of approximately 2.47 in Layer 1 indicates near-saturation conditions (Wathelet et al., 2004).

5. Discussion

The results collectively affirm and extend established theoretical predictions regarding surface wave propagation in layered elastic media. The dispersion data in Table 1 demonstrate with quantitative precision that phase velocity is a monotonically decreasing function of frequency in normally dispersive systems a direct consequence of longer wavelengths penetrating deeper, stiffer material while shorter wavelengths remain confined to shallow, softer layers. This fundamental property, originally formalized by Haskell (1953) and exploited by Park et al. (1999) through the MASW framework, underpins every surface wave-based geophysical investigation. The inversion performance documented in Table 2 is particularly significant. Mean absolute errors below 1.3% across all five layers demonstrate that least-squares Rayleigh wave inversion, when applied to high-quality dispersion data in normally dispersive sites, achieves near-borehole accuracy without subsurface drilling. This finding reinforces Xia et al.'s (1999) original claim and validates the application of MASW for cost-effective Vs30 estimation across India's rapidly urbanizing geotechnical landscape, where borehole data density remains inadequate relative to infrastructure development demands (Xia et al., 1999; Aki & Richards, 2002).

The method comparison in Table 3 reveals a clear performance hierarchy correlated with operational complexity. MASW emerges as the optimal balance between accuracy and feasibility for engineering applications, while full-waveform inversion is positioned as the high-precision option for research-grade site characterization. This hierarchy aligns with recommendations in Socco and Strobbia (2004) and Foti et al. (2015), and carries practical implications for geotechnical investigations in India, where resource constraints often necessitate economical field acquisition strategies. ReMi's passive approach, while sacrificing resolution, enables site characterization in urban environments where active seismic sources are impractical. Table 4 introduces Love waves into the comparative analysis, revealing that velocity contrasts between stiff and soft sites exceed 40% across the entire measured frequency band. This substantial contrast directly influences site amplification factors in seismic design codes. The stronger dispersion amplitude observed in soft alluvial sites (Model A) indicates a more complex frequency-dependent response, requiring multi-frequency inversion strategies rather than single-frequency approximations. Tokimatsu et al. (1992) previously identified that ignoring this dispersion complexity leads to systematic underestimation of Vs in shallow layers a finding corroborated here by the Model A velocity profile's steep dispersion gradient between 2 and 20 Hz.

The multi-mode inversion results in Table 5 demonstrate that inclusion of higher-mode dispersion information yields well-constrained elastic parameter estimates with Poisson's ratios physically consistent with saturated alluvium, and RMS misfits below 0.025. The smooth Vs gradient from 185 m/s to 890 m/s across 20 m confirms a normally dispersive stratigraphy amenable to stable inversion. In contrast, sites with velocity reversals where a soft layer underlies a stiffer formation would require global search inversion algorithms such as the neighborhood algorithm implemented by Wathelet et al. (2004) to avoid entrapment in local misfit minima. Taken together, these results confirm the study's hypothesis: increasing impedance contrast amplifies higher-mode Rayleigh wave contributions, and mode misidentification in such settings constitutes a primary source of inversion error. Boaga et al. (2013) attributed mode confusion specifically to ambiguity in ellipticity characterization, and Luo et al. (2009) demonstrated that high-resolution Radon transform mode separation substantially mitigates this issue. Future investigations should integrate both strategies for sites with complex stratigraphy. Furthermore, Biot's (1956) poroelastic framework suggests that fluid saturation prevalent in Indian lowland alluvial zones modifies surface wave velocity beyond purely elastic predictions, a factor not fully captured in classical Thomson-Haskell modeling and warranting incorporation into next-generation inversion algorithms. Passive ambient noise methods described by Campillo and Paul (2003) and Shapiro and Campillo (2004) offer complementary capabilities for imaging deeper velocity structure beyond active-source MASW's penetration limits, presenting a natural extension of the methodology presented here.

7. Conclusion

This study has demonstrated that surface wave propagation in layered materials is governed by a deterministic relationship between frequency, wavelength, and depth-dependent shear-wave velocity, with dispersion curve morphology serving as a precise fingerprint of subsurface elastic structure. Multi-mode Rayleigh wave inversion consistently outperforms single-mode approaches, particularly at sites with significant impedance contrasts where higher modes carry substantial energy. Inversion accuracy below 1.3% mean error validates the reliability of surface wave methods for near-surface Vs profiling. Love wave analysis adds complementary sensitivity to horizontal velocity heterogeneity. For Indian geotechnical practice, integration of MASW with neighborhood algorithm inversion and multi-mode extraction represents the most scientifically robust and operationally viable strategy for seismic site characterization. Further development should address poroelastic layer effects and real-time multi-mode field processing.

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