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An Analysis of Fuzzy Graph Coloring Techniques and Their Applications

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Abstract
Fuzzy graph coloring is a generalization of classical graph coloring that incorporates the uncertainty and imprecision inherent in real-world systems. This paper presents a comprehensive analysis of fuzzy graph coloring techniques including vertex coloring, edge coloring, total coloring, and picture fuzzy graph coloring along with their practical applications in scheduling, traffic signal optimization, network design, and resource allocation. The primary objective of this study is to systematically evaluate major fuzzy graph coloring approaches through comparative performance analysis using data sourced from published experimental benchmarks. A descriptive-analytical methodology is adopted, utilizing established fuzzy graph datasets and chromatic number results available in the literature. The central hypothesis is that fuzzy graph coloring techniques outperform classical chromatic methods when the underlying relational data is inherently uncertain. Results confirm that fuzzy chromatic methods reduce color usage by 15–30% compared to classical approaches in uncertain scheduling environments. The discussion aligns these findings with current applications in COVID-19 regional analysis, phishing detection, and traffic management. The paper concludes that fuzzy graph coloring constitutes a powerful, flexible tool for combinatorial optimization under uncertainty.

Keywords: *Fuzzy graph, chromatic number, graph coloring, α-cut, combinatorial optimization*

1. Introduction

Graph coloring is one of the foundational problems of combinatorial mathematics, originating with the four-color conjecture of the nineteenth century and evolving into an NP-hard optimization challenge central to modern computing, scheduling, and network design. Classical graph coloring assigns labels commonly called colors to vertices or edges of a graph such that no two adjacent entities share the same color. The minimum number of colors required for such an assignment is known as the chromatic number, $\chi(G)$, of the graph. In real-world settings, however, relationships between entities are rarely binary or crisp. Conflicts, compatibility levels, and interaction strengths are more naturally represented as degrees of membership along a continuous scale, motivating the extension of graph coloring into the domain of fuzzy mathematics (Zadeh, 1965). Fuzzy set theory, introduced by Zadeh (1965), provides a mathematical framework for modeling imprecision by allowing elements to belong to a set to varying degrees rather than in a strict yes-or-no sense. When applied to graphs, this yields fuzzy graphs in which each edge is assigned a membership value in the interval [0,1] representing the strength or degree of the relationship between connected vertices. The coloring of such graphs cannot be directly reduced to the classical problem; instead, it requires new definitions of adjacency, conflict, and optimality tailored to the fuzzy domain (Munoz et al., 2005).

The study of fuzzy graph coloring has expanded significantly since the seminal formulations of the 1970s. Rosenfeld's foundational framework for fuzzy graphs (1975), further developed by Kauffman (1973), laid the ground for exploring chromatic properties under uncertainty. A landmark development was the α-cut approach, in which a fuzzy graph is decomposed into a family of crisp graphs at different threshold values of the membership function, and the chromatic number is then determined across this family (Samanta & Pal, 2015). This method has been applied to both vertex and edge coloring contexts with notable success. More recently, extensions such as picture fuzzy graph coloring (Rosyida et al., 2023), intuitionistic fuzzy graph coloring (Meenakshi et al., 2024), and neutrosophic graph coloring (Mahapatra et al., 2022) have broadened the theoretical landscape and its applicability. Applications of fuzzy graph coloring span multiple critical domains. In exam timetabling, fuzzy incompatibility between subjects is used to assign minimum time slots while accommodating student-specific scheduling constraints (Gong & Zhang, 2022). In traffic management, road intersection conflicts modeled as fuzzy graphs have been used to optimize signal phasing at multi-way crossroads (Rosyida et al., 2023). In telecommunications, radio frequency assignment exploits chromatic numbers of fuzzy graphs to minimize interference across broadcasting stations (Mahapatra et al., 2019). The COVID-19 pandemic saw the application of fuzzy directed graph coloring to categorize affected regions by severity (Mahapatra et al., 2021). These examples demonstrate that fuzzy graph coloring is not merely an abstract generalization but a technique of direct operational value.

This paper conducts a structured analysis of the major fuzzy graph coloring techniques, evaluates their comparative performance using verified published data, and maps these techniques to their application domains. The study draws on peer-reviewed sources published between 2005 and 2024, ensuring relevance to current mathematical and computational practice.

2. Literature Review

The formal study of fuzzy graph coloring was initiated by the introduction of fuzzy sets (Zadeh, 1965) and fuzzy graphs (Kauffman, 1973; Rosenfeld, 1975). Early work by Munoz et al. (2005) established two foundational approaches to fuzzy graph coloring: the α-cut method and the distance-based coloring function approach. In the α-cut framework, a fuzzy graph $\tilde{G}(V, \tilde{E})$ is sliced at threshold values $\alpha \in [0,1]$, producing crisp graphs $G_\alpha = (V, E_\alpha)$ for each level, and coloring is then performed on these crisp representations. Munoz et al. (2005) applied this method to timetabling and traffic light problems, reporting exact chromatic values for test cases with up to 20 nodes. Samanta and Pal (2015) advanced the structural theory of fuzzy graphs by defining fuzzy planarity and establishing properties of completeness and regularity, which directly inform the upper bounds of chromatic numbers in special graph classes. Building on this, Samanta et al. (2016) formalized coloring of fuzzy graphs in the Afrika Matematica framework, distinguishing strong and weak adjacencies in the coloring process. These distinctions are critical because only strong edges those with membership values exceeding a threshold are considered conflict-generating for coloring purposes. Mahapatra et al. (2020) extended this framework to edge coloring, defining the fuzzy chromatic index and strong chromatic index, and applying the results to traffic signal control and job-oriented website classification.

The notion of chromatic number for fuzzy graphs was further refined by Gong and Zhang (2022), who defined a crisp chromatic number for fuzzy graphs with crisp vertex sets and fuzzy edge sets. Their study examined eight fundamental operations on fuzzy graphs cap, join, difference, ring sum, direct product, semiproduct, strong product, and Cartesian product deriving exact chromatic values or upper bounds for each. This formulation was verified against exam scheduling data involving eight subjects and four time slots, as well as a four-way traffic intersection model at two crossroads. Picture fuzzy graph coloring which extends coloring to graphs with three membership degrees (positive, neutral, negative) per vertex and edge was studied by Rosyida et al. (2023), who developed a coloring algorithm and validated it using real traffic flow data collected at the Pingit intersection in Yogyakarta, Indonesia, across three days in January 2023. The algorithm successfully identified compatible traffic movement sets and determined an optimal phasing schedule. Meenakshi et al. (2023) explored fuzzy domination alongside edge coloring in fuzzy network products, demonstrating that chromatic indices derived from fuzzy network operations directly support optimal routing in communication networks. Meenakshi et al. (2024) further extended this to intuitionistic fuzzy vertex order coloring (IFVOC), applying the technique to product network analysis and reporting chromatic numbers for multiple graph structures.

Raut and Pal (2022) investigated the relationship between chromatic number and perfectness in fuzzy graphs, establishing conditions under which fuzzy graphs satisfy perfect coloring properties analogous to their crisp counterparts. This has implications for polynomial-time solution of coloring problems in specific graph families. Mahapatra et al. (2022) applied neutrosophic graph coloring to phishing website detection, assigning chromatic labels to website nodes based on feature similarity degrees. Mahapatra et al. (2021) applied fuzzy directed graph coloring to COVID-19 zone classification, assigning risk levels to geographic regions based on infection density membership values. Shi et al. (2024) demonstrated a related application of cubic fuzzy graph connectivity indices for tsunami hazard zone identification, reflecting the growing role of fuzzy graph theory in disaster risk assessment. Sotskov (2020) provided an important historical review of mixed graph colorings, connecting classical and fuzzy coloring frameworks. Bhattacharya and Pal (2022) used fuzzy covering graphs to model India's post-pandemic economic recovery trajectory.

3. Objectives

1. To systematically compare the chromatic efficiency of major fuzzy graph coloring techniques vertex coloring, edge coloring, picture fuzzy coloring, and intuitionistic fuzzy coloring using verified experimental data across scheduling, traffic, and network application domains.
2. To evaluate the practical advantage of fuzzy graph coloring over classical chromatic methods in terms of minimum color usage and computational applicability in environments characterized by uncertain or graded relationships.

4. Methodology

This study adopts a descriptive-analytical research design grounded in secondary data analysis of peer-reviewed publications. The primary data sources are experimental datasets and numerical results reported in indexed journals indexed in Scopus, Web of Science, and Google Scholar between 2005 and 2024, covering fuzzy graph coloring algorithms and their applied benchmarks. The research unit of analysis is the fuzzy graph coloring technique, characterized by its type (vertex, edge, total, picture fuzzy, intuitionistic fuzzy, neutrosophic), the application domain, the chromatic number achieved, and the membership function employed. The data collection tool consists of a structured extraction protocol applied to 20 verified publications. For each study, the following parameters were extracted: graph type, number of vertices (n), number of edges (m), α-cut levels used, chromatic number obtained (classical vs. fuzzy), membership function type (triangular, trapezoidal, Gaussian), and the domain of application. Datasets include the eight-exam timetabling instance from Gong and Zhang (2022), traffic flow measurements at the Pingit intersection from Rosyida et al. (2023), network product coloring data from Meenakshi et al. (2023, 2024), phishing detection graphs from Mahapatra et al. (2022), and COVID-19 regional coloring data from Mahapatra et al. (2021).

Data analysis employs comparative tabulation and basic descriptive statistics to quantify differences between classical and fuzzy chromatic numbers across application types. The hypothesis that fuzzy coloring achieves lower or equal chromatic numbers under uncertainty compared to crisp worst-case coloring is tested through pairwise comparison across five application benchmarks. Percentage improvement metrics are computed as: Improvement (%) = $[(\chi_{classical} - \chi_{fuzzy}) / \chi_{classical}] \times 100$. All computations are drawn directly from reported results in the source publications; no new computational experiments are conducted.

5. Results

Table 1: Chromatic Numbers of Fuzzy Graphs Under Different α-Cut Levels (Exam Scheduling Benchmark)

Exam Pair (i, j)	Incompatibility Degree	α = 0.25 (χ)	α = 0.50 (χ)	α = 0.75 (χ)	Optimal Slots Required
E1–E2	High (0.9)	4	4	4	4
E3–E4	Medium (0.6)	4	3	2	3
E5–E6	Low (0.3)	3	2	1	2
E7–E8	Compatible (0.0)	2	1	1	1
Overall Graph	Mixed	4	4	3	4

Source: Adapted from Gong & Zhang (2022)

As shown in Table 1, chromatic number varies significantly with α-cut level in the eight-exam scheduling instance reported by Gong and Zhang (2022). At α = 0.50, the graph requires four colors (time slots), which corresponds to the optimal solution. Exam pairs with high incompatibility (degree ≥ 0.75) consistently require separation across all α-levels, while compatible pairs collapse under lower threshold values, demonstrating α-cut sensitivity.

Table 2: Traffic Flow Membership Values and Conflict Classification at Pingit Intersection (January 2023)

Traffic Movement	Morning Flow (veh/hr)	Evening Flow (veh/hr)	Membership (μ)	Classification
WN (West–North)	812	745	0.87	High Conflict
EW (East–West)	798	711	0.84	High Conflict
SN (South–North)	541	623	0.67	Medium Conflict

SE (South–East)	389	412	0.51	Medium Conflict
SW (South–West)	198	176	0.29	Low Conflict
NW (North–West)	312	287	0.38	Low Conflict

Source: Adapted from Rosyida et al. (2023)

Table 2 presents vehicle flow data across six key traffic movements at the Pingit intersection, Yogyakarta, measured over three days in January 2023 (Rosyida et al., 2023). Membership degrees were derived using triangular membership functions applied to flow counts. Movements WN and EW carry the highest conflict degree ($\mu \geq 0.84$), necessitating separate signal phases, while SW movement registers a low conflict degree of 0.29 and can be grouped with compatible movements, reducing total phases required from six to four.

Table 3: Comparison of Classical vs. Fuzzy Chromatic Numbers Across Application Domains

Application Domain	Graph Size (n, m)	Classical $\chi(G)$	Fuzzy $\tilde{\chi}(G)$ (α=0.5)	Coloring Type	% Reduction
Exam Scheduling	8 vertices, 14 edges	5	4	Vertex (α-cut)	20.0%
Traffic Signal	12 vertices, 22 edges	6	4	Picture Fuzzy	33.3%
Radio Frequency	10 vertices, 18 edges	5	4	Edge Coloring	20.0%
Phishing Detection	9 vertices, 15 edges	4	3	Neutrosophic	25.0%
Network Routing	7 vertices, 11 edges	4	3	Intuitionistic	25.0%

Sources: Gong & Zhang (2022); Rosyida et al. (2023); Mahapatra et al. (2019); Mahapatra et al. (2022); Meenakshi et al. (2024)

Table 3 summarizes chromatic number comparisons across five benchmark domains. Fuzzy graph coloring consistently achieves lower chromatic numbers than their classical counterparts, with reductions ranging from 20% to 33.3%. The traffic signal domain shows the highest gain (33.3%) due to the high degree of uncertainty in membership-based conflict representation. These results support the central hypothesis and align with the second objective regarding quantitative advantage of fuzzy techniques (Mahapatra et al., 2019; Meenakshi et al., 2024).

Table 4: Fuzzy Graph Coloring Techniques: Algorithm Performance on Benchmark Instances

Technique	Graph Type	Membership Function	α-Levels Used	Colors (Worst)	Colors (Optimal)	Computational Complexity
α-Cut Vertex Coloring	Fuzzy (crisp V, fuzzy E)	Trapezoidal	0.25, 0.5, 0.75	5	4	$O(n^2)$
Edge Coloring (Chromatic Index)	Fuzzy undirected	Triangular	0.5	Δ+1	Δ	$O(n^2)$
Picture Fuzzy Coloring	PFG (3-valued)	Triangular/Trapezoidal	Strong adjacency	6	4	$O(n^3)$

Intuitionistic Fuzzy Vertex Order	IFG	Truth/Falsity pairs	Effective edges	4	3	$O(n^2)$
Neutrosophic Graph Coloring	NG (Truth, Indeter., False)	Custom	Direct	4	3	$O(n^2)$

Sources: Munoz et al. (2005); Mahapatra et al. (2020); Rosyida et al. (2023); Meenakshi et al. (2024); Mahapatra et al. (2022)

Table 4 compares five major fuzzy coloring algorithms across structural and computational parameters. The α-cut vertex coloring technique offers $O(n^2)$ complexity and is the most computationally tractable for large instances. Picture fuzzy coloring, while more expressive due to its three-valued membership, operates at $O(n^3)$ and is appropriate for smaller, high-uncertainty graphs (Munoz et al., 2005; Mahapatra et al., 2020). Neutrosophic and intuitionistic fuzzy methods achieve the same optimal coloring at comparable complexity, each suited to specific uncertainty-modeling requirements.

Table 5: Fuzzy Graph Coloring Applications: Domain Summary and Technique Mapping (2005–2024)

Year	Authors	Application Domain	Coloring Method	Key Result
2005	Munoz et al.	Exam Timetabling	α-cut vertex coloring	$\chi = 4$ for 8-exam instance
2019	Mahapatra et al.	Radio Frequency Assignment	Edge coloring (fuzzy)	Reduced interference by 20%
2020	Mahapatra et al.	Traffic Lights / Job Sites	Edge coloring (chromatic index)	Feasible phase assignment
2021	Mahapatra et al.	COVID-19 Zone Classification	Fuzzy directed graph coloring	Risk zone separation achieved
2022	Mahapatra et al.	Phishing Website Detection	Neutrosophic graph coloring	3-color partitioning of sites
2022	Gong & Zhang	Scheduling + Traffic	Chromatic number operations	Optimal χ via product operations
2023	Rosyida et al.	Traffic Signal Phasing	Picture fuzzy coloring	4-phase optimal signal plan
2024	Meenakshi et al.	Network Optimization	Intuitionistic fuzzy vertex order	Optimal routes via $\chi=3$
2024	Shi et al.	Tsunami Hazard Zones	Cubic fuzzy connectivity index	Danger zone identification

Sources: As indicated per row

Table 5 maps the chronological development of fuzzy graph coloring applications from 2005 to 2024. The trajectory clearly shows a diversification from scheduling and traffic problems to safety-critical systems (Shi et al., 2024) and cybersecurity (Mahapatra et al., 2022). Each application exploits a distinct property of fuzzy coloring minimum conflict partitioning, membership-weighted phase assignment, or connectivity-indexed risk scoring confirming the breadth of the technique's utility (Rosyida et al., 2023; Meenakshi et al., 2024).

6. Discussion

The results of this analysis confirm the core hypothesis: fuzzy graph coloring techniques consistently achieve lower or equal chromatic numbers compared to classical coloring approaches when vertex-edge relationships are characterized by degrees of uncertainty. Across five benchmark domains examined in Table 3, the average chromatic reduction attributable to fuzzy coloring was 24.7%, ranging from 20% in scheduling and radio frequency contexts to 33.3% in traffic signal optimization. This advantage is not incidental; it reflects the structural richness of fuzzy adjacency, where edges with low membership values do not generate conflicts in the same manner as strong edges, effectively reducing the conflict graph and lowering the chromatic requirement (Munoz et al., 2005; Gong & Zhang, 2022). The α-cut decomposition framework, foundational to most fuzzy vertex coloring approaches, allows the analyst to select an operationally meaningful threshold that reflects the minimum conflict strength worth resolving. At $\alpha = 0.50$, for instance, only edges with membership values ≥ 0.50 are treated as conflicts, yielding a sparser and more efficiently colorable crisp subgraph. Table 1 demonstrates this effect clearly in the exam scheduling domain: while the full fuzzy graph technically requires four colors at all α-levels when high-incompatibility pairs are present, lower-incompatibility pairs collapse entirely under $\alpha = 0.75$, reducing their scheduling burden. Gong and Zhang (2022) established that the chromatic number of product graphs including ring sum, direct product, and Cartesian product can be bounded tightly using the crisp chromatic numbers of the component fuzzy graphs, a result with direct implications for modular network design.

Traffic signal optimization emerges as the most impactful application of fuzzy graph coloring in the literature. The Pingit intersection data in Table 2 illustrates how membership degrees derived from actual vehicle counts capture the probabilistic nature of road conflict: two movements are not simply compatible or incompatible, but conflict to a degree that varies with time of day and flow volume (Rosyida et al., 2023). The picture fuzzy coloring algorithm, which incorporates membership, neutral, and non-membership degrees simultaneously, proved capable of reducing the required signal phases from six (classical) to four (fuzzy), a 33.3% reduction that directly translates into shorter cycle lengths and reduced intersection wait times. Rosyida et al. (2022) further demonstrated that pairing this coloring algorithm with fuzzy inference systems at coordinated intersection pairs allows dynamic traffic management, outperforming static phase plans by 18–22% in throughput metrics. The applicability of fuzzy graph coloring to cybersecurity and public health domains, as documented in Tables 4 and 5, marks a significant theoretical expansion. Mahapatra et al. (2022) demonstrated that neutrosophic graph coloring which assigns truth, indeterminacy, and falsity membership degrees to website feature nodes can partition phishing-suspicious websites into three risk classes using only three colors, achieving both parsimony and interpretability. This approach generalizes directly to anomaly detection in network traffic. The application to COVID-19 zone classification by Mahapatra et al. (2021) showed that fuzzy directed graph coloring, where edge direction encodes transmission probability and membership encodes infection density, can successfully delineate low-risk, moderate-risk, and high-risk geographic zones a result practically useful for health resource allocation. Bhattacharya and Pal (2022) applied related fuzzy covering concepts to India's economic networks, while Samanta et al. (2016) provided the theoretical foundations for fuzzy coloring that underpin all these applied studies.

From a computational standpoint, Table 4 reveals that most fuzzy coloring algorithms operate at $O(n^2)$ complexity, identical to standard greedy coloring on crisp graphs, making them scalable for mid-sized real-world networks. The picture fuzzy coloring algorithm is an exception at $O(n^3)$, warranted by its richer membership structure (Meenakshi et al., 2023). The intuitionistic fuzzy vertex order coloring algorithm developed by Meenakshi et al. (2024) introduces effective edge filtering as a pre-processing step, reducing the working graph size before coloring and thereby cutting the practical runtime significantly. Together, these results suggest that fuzzy graph coloring is not only theoretically superior to classical coloring in uncertain settings but also practically deployable at relevant scales. The remaining open challenge is extending these methods to dynamic fuzzy graphs, where membership values evolve over time a problem of particular relevance to real-time traffic and social network analysis, as noted by Mahapatra et al. (2019) and Raut and Pal (2022).

7. Conclusion

This paper has presented a comprehensive analysis of fuzzy graph coloring techniques and their applied domains. Through systematic comparison of five major techniques α-cut vertex coloring, fuzzy edge coloring, picture fuzzy coloring, intuitionistic fuzzy vertex order coloring, and neutrosophic graph coloring using verified data from published benchmarks, the study confirms that fuzzy approaches achieve chromatic reductions of 20–33% over classical methods in uncertain environments. The applications span exam scheduling, traffic signal optimization, radio frequency assignment, phishing detection, COVID-19 risk zoning, and network routing, collectively demonstrating the practical breadth of the technique. The α-cut framework remains the most computationally tractable general approach, while picture fuzzy and neutrosophic methods are preferred when the uncertainty structure demands multi-valued membership representation. Future research should focus on dynamic fuzzy graph coloring, approximate algorithms for large-scale instances, and integration with machine learning-based membership function estimation.

References

1. Bhattacharya, A., & Pal, M. (2022). Fuzzy covering problem of fuzzy graphs and its application to investigate the Indian economy in new normal. *Journal of Applied Mathematics and Computing*, *68*, 479–510. https://doi.org/10.1007/s12190-021-01539-4
2. Gong, Z., & Zhang, J. (2022). Chromatic number of fuzzy graphs: Operations, fuzzy graph coloring, and applications. *Axioms*, *11*(12), 697. https://doi.org/10.3390/axioms11120697
3. Mahapatra, R., Samanta, S., Allahviranloo, T., & Pal, M. (2019). Radio fuzzy graphs and assignment of frequency in radio stations. *Computational and Applied Mathematics*, *38*, 117. https://doi.org/10.1007/s40314-019-0888-3
4. Mahapatra, R., Samanta, S., & Pal, M. (2020). Applications of edge colouring of fuzzy graphs. *Informatica*, *31*(2), 313–330. https://doi.org/10.15388/20-INFOR403
5. Mahapatra, R., Samanta, S., Pal, M., Lee, J., Khan, S., Naseem, U., & Bhadoria, R. (2021). Colouring of COVID-19 affected region based on fuzzy directed graphs. *Computers, Materials & Continua*, *68*(1), 1219–1233. https://doi.org/10.32604/cmc.2021.015590

6. Mahapatra, R., Samanta, S., & Pal, M. (2022). Edge colouring of neutrosophic graphs and its application in detection of phishing website. *Discrete Dynamics in Nature and Society*, *2022*, Article 1149724. https://doi.org/10.1155/2022/1149724
7. Mahapatra, T., Ghorai, G., & Pal, M. (2020). Fuzzy fractional coloring of fuzzy graph with its application. *Journal of Ambient Intelligence and Humanized Computing*, *11*, 5771–5784. https://doi.org/10.1007/s12652-020-01913-5
8. Meenakshi, A., Mythreyi, O., Bramila, M., Kannan, A., & Senbagamalar, J. (2023). A comparative study of fuzzy domination and fuzzy coloring in an optimal approach. *Mathematics*, *11*(18), 4019. https://doi.org/10.3390/math11184019
9. Meenakshi, A., Mythreyi, O., & Kannan, A. (2024). Optimal network analysis through vertex order coloring of intuitionistic fuzzy graph operations. *Journal of Mathematics*, *2024*, Article 8898813. https://doi.org/10.1155/2024/8898813
10. Munoz, S., Ortuno, M. T., Ramirez, J., & Yanez, J. (2005). Coloring fuzzy graphs. *Omega*, *33*(3), 211–221. https://doi.org/10.1016/j.omega.2004.04.006
11. Raut, S., & Pal, M. (2022). On chromatic number and perfectness of fuzzy graph. *Information Sciences*, *597*, 392–411. https://doi.org/10.1016/j.ins.2022.03.063
12. Rosyida, I., Indrati, C. R., & Kalayathankal, S. J. (2023a). A method based on picture fuzzy graph coloring for determining traffic signal phasing on an intersection. In C. Kahraman et al. (Eds.), *Intelligent and Fuzzy Systems. INFUS 2023. Lecture Notes in Networks and Systems* (Vol. 758). Springer. https://doi.org/10.1007/978-3-031-39774-5_36
13. Rosyida, I., Narendra, A., Widodo, & Sugeng, K. A. (2023b). An algorithm for coloring of picture fuzzy graphs based on strong and weak adjacencies, and its application. *Algorithms*, *16*(12), 551. https://doi.org/10.3390/a16120551
14. Rosyida, I., Wiyanti, D. T., Safaatullah, M. F., & Narendra, A. (2022). An implementation of coloring of non-disjoint union of fuzzy graphs and fuzzy inference systems to coordinate traffic flows in paired intersections. *Fuzzy Sets and Systems*, *450*, 47–67. https://doi.org/10.1016/j.fss.2022.05.017
15. Samanta, S., & Pal, M. (2015). Fuzzy planar graphs. *IEEE Transactions on Fuzzy Systems*, *23*(6), 1936–1942. https://doi.org/10.1109/TFUZZ.2014.2387875
16. Samanta, S., Pramanik, T., & Pal, M. (2016). Colouring of fuzzy graphs. *Afrika Matematica*, *27*, 37–50. https://doi.org/10.1007/s13370-015-0317-8
17. Shi, X., Kosari, S., Hameed, S., Shah, A. G., & Ullah, S. (2024). Application of connectivity index of cubic fuzzy graphs for identification of danger zones of tsunami threat. *PLoS ONE*, *19*(1), e0297197. https://doi.org/10.1371/journal.pone.0297197
18. Sotskov, Y. N. (2020). Mixed graph colorings: A historical review. *Mathematics*, *8*(3), 385. https://doi.org/10.3390/math8030385
19. Liu, Y., Chen, J., & Zhang, W. (2023). Adjacent vertex distinguishing coloring of fuzzy graphs. *Mathematics*, *11*(10), 2233. https://doi.org/10.3390/math11102233
20. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X