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An Integrated Inventory Model with Stock Dependent Demand under Inflation

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Abstract

This study develops an integrated inventory model for a two-echelon supply chain, incorporating stock-dependent demand, inflation, partial backlogging, and product deterioration, with both forward and reverse manufacturing processes. The model simultaneously addresses retailer and supplier operations, including ordering, holding, deterioration, backordering, lost sales, and production costs, along with remanufacturing, recycling, and life-cycle design expenses. A numerical example demonstrates the application of the model using realistic cost and operational parameters, revealing optimal retailer cycle times, supplier production rates, remanufacturing rates, and total supply chain costs. Sensitivity analyses were conducted for key parameters, including retailer cost coefficients (c_1r, c_2r), demand intensity (b), and production cost (c), showing that increases in cost parameters and demand generally elevate total system cost, while cycle-time components respond moderately to these variations. The results provide actionable insights for optimizing inventory policies, shipment schedules, and remanufacturing strategies under economic and operational uncertainties. Overall, the proposed framework offers a practical, cost-efficient decision-making tool for managing inventory and enhancing sustainability in complex supply chains.

Keywords: *Integrated Inventory Model, Stock-Dependent Demand, Inflation Impact, Reverse and Forward Manufacturing, Supply Chain Cost Optimization.*

1. Introduction

Inventory management plays a critical role in ensuring operational efficiency and cost-effectiveness in supply chains, particularly under dynamic market conditions such as inflation and stock-dependent demand. Traditional inventory models, including the classic Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) frameworks, have been extensively studied for optimizing order sizes and minimizing holding and setup costs (Ghare, 1963; Porteus, 1986; Goyal, 1985; Teng, 2002). However, these models often assume constant demand, neglecting practical scenarios where demand fluctuates with stock levels, market trends, and pricing strategies (Min, Zhou, & Zhao, 2010; Singh & Singh, 2017). To address such complexities, researchers have introduced integrated inventory models that incorporate deteriorating items, imperfect production, and backlogging, offering more realistic frameworks for decision-making (Salameh & Jaber, 2000; Handa, Singh, & Katariya, 2021; Sarkar, Saren, & Cárdenas-Barrón, 2015). Moreover, trade credit policies, permissible delays in payments, and collaborative investment strategies have been shown to significantly influence inventory behavior and ordering decisions (Shah, 2006; Ouyang, Chen, & Yang, 2013; Teng, Yang, & Chern, 2013). The impact of external factors, such as inflation, on multi-echelon supply chains further complicates inventory planning, affecting both cost parameters and demand patterns (Padiyar et al., 2022; Handa, Singh, & Katariya, 2024). Considering these advancements, there is a clear need for an integrated inventory model that not only captures stock-dependent demand but also accounts for inflationary effects, partial backlogging, and variable holding costs, providing a comprehensive tool for sustainable and cost-efficient inventory management. Such a model can bridge the gap between theoretical research and practical application, enabling firms to optimize inventory policies in volatile economic environments while maintaining service levels and minimizing total costs (Tayal et al., 2021; Handa et al., 2021).

2. Formulations and Analysis of the Model

This section is divided into two distinct cases: the first presents the inventory model formulated for the retailer, and the second focuses on the corresponding model developed for the supplier.

Case I: Inventory Model for the Retailer during the First Cycle ($0 \le t \le T$)

At the beginning of the cycle, specifically at $t=0$, the retailer receives the delivery and immediately begins selling the items under a stock-dependent demand environment. The retailer's inventory level declines over time due to the combined effect of demand consumption and product deterioration. When the system reaches time $t=t_1$, the on-hand inventory becomes depleted, initiating a shortage period that continues throughout the interval $[t_1,T]$. The retailer ends up experiencing the most severe shortages at the end of cycle, when $t=T$. This mode of operation is demonstrated in Figure 1 and is mathematically described by differential equations describing inventory exhaustion and backlog creation.

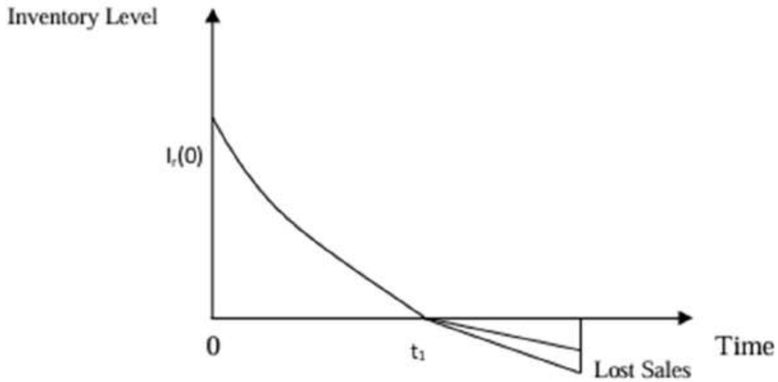

Figure 1: Retailer's inventory model

$$I'_r(t) = -\left(a + bI_r(t)\right) - \theta t I_r(t), \quad 0 \le t \le t_1 \tag{4.1}$$

$$I'_r(t) = -ae^{-\eta(T-t)}, \quad t_1 \le t \le T \tag{4.2}$$

By solving equations (4.1) and (4.2) under the boundary condition $I_r(t)=0$ at the specified time, the following results are obtained:

$$I_r(t) = a\left[\left(t_1 + \frac{bt_1^2}{2} + \frac{\theta t_1^3}{6}\right) - \left(t + \frac{bt^2}{2} + \frac{\theta t^3}{6}\right)\right]e^{-\left(bt + \frac{\theta t^2}{2}\right)}, \quad 0 \le t \le t_1 \tag{4.3}$$

$$I_r(t) = -\frac{ae^{-\eta T}}{\eta}\left(e^{\eta t} - e^{\eta t_1}\right), \ t_1 \le t \le T \tag{4.4}$$

The inventory level at time t=0, denoted as $I_r(0)$, can be expressed as follows:

$$I_r(0) = a\left(t_1 + \frac{bt_1^2}{2} + \frac{\theta t_1^3}{6}\right), \tag{4.5}$$

Since the highest level of shortages is reached at t=T, we can express the condition as follows:

$$I_r(T) = -\frac{ae^{-\eta T}}{\eta}\left(e^{\eta T} - e^{\eta t_1}\right), \tag{4.6}$$

The retailer's order quantity is determined as follows:

$$q = I_r(0) + \left(-I_r(T)\right) = a\left(t_1 + \frac{bt_1^2}{2} + \frac{\theta t_1^3}{6}\right) + \frac{ae^{-\eta T}}{\eta}\left(e^{\eta T} - e^{\eta t_1}\right), \tag{4.7}$$

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1. The holding cost incurred during the retailer's first cycle is given by:

$$HC_{r1} = c_{1r}\int_0^{t_1} I_r(t)e^{-rt}dt$$

$$HC_{r1} = c_{1r}a\left[\frac{e^{-rt_1}}{r^6}\left\{r^4 - br^3 - b^2r^4t_1^2 - 3b^2r^3t_1 - 3b^2r^2 - \theta r^3t_1 - 2r^2\theta - b\theta r^4t_1^3 - \frac{9b\theta r^3t_1^2}{2} - 10b\theta r^2t_1\right.\right.$$
$$\left.-10b\theta r - \frac{\theta^2r^4t_1^4}{4} - \frac{5\theta^2r^3t_1^4}{3} - 5\theta^2r^2t_1^2 - 10\theta^2rt_1 - 10\theta^2 + \frac{\theta^2r^3t_1^3}{6}\right\}$$
$$\left.+\frac{1}{r^6}\left(br^3 - r^4 + 3b^2r^2 + 2r^2\theta + 10b\theta r + 10\theta^2\right) - \frac{1}{r^3}\left(t_1 + \frac{bt_1^2}{2} + \frac{\theta t_1^3}{6}\right)\left(\theta + br - r^2\right)\right] \tag{4.8}$$

2. The holding cost for the retailer over n complete cycles is expressed as:

$$HC_r = \sum_{j=1}^{n} HC_{r1}e^{-r(j-1)T}$$

$$HC_r = c_{1r}a\left[\frac{e^{-rt_1}}{r^6}\left\{r^4 - br^3 - b^2r^4t_1^2 - 3b^2r^3t_1 - 3b^2r^2 - \theta r^3t_1 - 2r^2\theta - b\theta r^4t_1^3 - \frac{9b\theta r^3t_1^2}{2} - 10b\theta r^2t_1\right.\right.$$
$$\left.-10b\theta r - \frac{\theta^2r^4t_1^4}{4} - \frac{5\theta^2r^3t_1^4}{3} - 5\theta^2r^2t_1^2 - 10\theta^2rt_1 - 10\theta^2 + \frac{\theta^2r^3t_1^3}{6}\right\}$$
$$\left.+\frac{1}{r^6}\left(br^3 - r^4 + 3b^2r^2 + 2r^2\theta + 10b\theta r + 10\theta^2\right) - \frac{1}{r^3}\left(t_1 + \frac{bt_1^2}{2} + \frac{\theta t_1^3}{6}\right)\left(\theta + br - r^2\right)\right]\left\{\frac{1-e^{-rnT}}{1-e^{-rT}}\right\} \tag{4.9}$$

3. The deterioration cost for the retailer during the first cycle is calculated as:

$$DC_{r1} = c_{2r}\int_0^{t_1} \theta t I_r(t)e^{-rt}dt$$

$$DC_{r1} = c_{2r}a\theta\left[\frac{e^{-rt_1}}{r^7}\left\{r^5t_1 + 2r^4 - br^4t_1 - 3br^3 - \theta r^4t_1^2 - 5\theta r^3t_1 - 8\theta r^2 - b^2r^5t_1^3 - 5b^2r^4t_1^2 - 12b^2r^3t_1\right.\right.$$
$$-12b^2r^2 - b\theta r^5t_1^4 - \frac{13b\theta r^4t_1^3}{2} - \frac{47b\theta r^3t_1^2}{2} - 50b\theta r^2t_1 - 50b\theta r - \frac{\theta^2r^5t_1^5}{4} - 60\theta^2rt_1 - 60\theta^2 + 3\theta r^4t_1^2$$
$$\left.-2\theta^2r^4t_1^4 - \frac{99\theta^2r^3t_1^3}{2} - 30\theta^2r^2t_1^2\right\} + \frac{1}{r^6}\left(3br^3 - 2r^4 + 8r^2\theta + 12b^2r^2 + 50b\theta r + 60\theta^2\right)$$
$$\left.+\frac{1}{r^4}\left(t_1 + \frac{bt_1^2}{2} + \frac{\theta t_1^3}{6}\right)\left(r^2 - 2br - 3\theta\right)\right] \tag{4.10}$$

4. The deterioration cost incurred by the retailer over n complete cycles is given by:

$$DC_r = \sum_{j=1}^{n} DC_{r1} e^{-r(j-1)T}$$

$$DC_r = c_{2r} a\theta \left[\frac{e^{-rt_1}}{r^7} \left\{ r^5 t_1 + 2r^4 - br^4 t_1 - 3br^3 - \theta r^4 t_1^2 - 5\theta r^3 t_1 - 8\theta r^2 - b^2 r^5 t_1^3 - 5b^2 r^4 t_1^2 - 12b^2 r^3 t_1 \right.\right.$$

$$- 12b^2 r^2 - b\theta r^5 t_1^4 - \frac{13b\theta r^4 t_1^3}{2} - \frac{47b\theta r^3 t_1^2}{2} - 50b\theta r^2 t_1 - 50b\theta r - \frac{\theta^2 r^5 t_1^5}{4} - 60\theta^2 r t_1 - 60\theta^2 + 3\theta r^4 t_1^2$$

$$\left. - 2\theta^2 r^4 t_1^4 - \frac{99\theta^2 r^3 t_1^3}{2} - 30\theta^2 r^2 t_1^2 \right\} + \frac{1}{r^6}\left(3br^3 - 2r^4 + 8r^2\theta + 12b^2 r^2 + 50b\theta r + 60\theta^2 \right)$$

$$\left. + \frac{1}{r^4}\left(t_1 + \frac{bt_1^2}{2} + \frac{\theta t_1^3}{6} \right)\left(r^2 - 2br - 3\theta \right) \right] \left\{ \frac{1 - e^{-mT}}{1 - e^{-rT}} \right\} \quad (4.11)$$

5. The backordering cost incurred during the retailer's first cycle is expressed as:

$$BC_{r1} = c_{3r} \int_{t_1}^{T} \left[-I_r(t) \right] e^{-rt} dt$$

$$BC_{r1} = \frac{c_{3r} a e^{-\eta T}}{\eta} \left[\frac{e^{-(r-\eta)T}}{(\eta - r)} - \frac{\eta e^{-(r-\eta)t_1}}{r(\eta - r)} + \frac{e^{\eta t_1 - rT}}{r} \right] \quad (4.12)$$

6. The backordering cost for the retailer across n complete cycles is given by:

$$BC_r = \sum_{j=1}^{n} BC_{r1} e^{-r(j-1)T}$$

$$BC_{r1} = \frac{c_{3r} a e^{-\eta T}}{\eta} \left[\frac{e^{-(r-\eta)T}}{(\eta - r)} - \frac{\eta e^{-(r-\eta)t_1}}{r(\eta - r)} + \frac{e^{\eta t_1 - rT}}{r} \right] \left\{ \frac{1 - e^{-rnT}}{1 - e^{-rT}} \right\} \quad (4.13)$$

7. The lost sales cost for the retailer during the first cycle is determined as:

$$LC_{r1} = c_{4r}\int_{t_1}^{T}\left[1-e^{-\eta(T-t)}\right]ae^{-rt}dt$$

$$LC_{r1} = c_{4r}a\left[\frac{\left(e^{-rt_1}-e^{-rT}\right)}{r}-\frac{\left(e^{-rT}-e^{-\eta T-(r-\eta)t_1}\right)}{(\eta-r)}\right] \quad (4.14)$$

8. The lost sales cost for the retailer over n complete cycles is given by:

$$LC_r = \sum_{j=1}^{n} LC_{r1}e^{-r(j-1)T}$$

$$LC_r = c_{4r}a\left[\frac{\left(e^{-rt_1}-e^{-rT}\right)}{r}-\frac{\left(e^{-rT}-e^{-\eta T-(r-\eta)t_1}\right)}{(\eta-r)}\right]\left\{\frac{1-e^{-rnT}}{1-e^{-rT}}\right\} \quad (4.15)$$

9. The ordering cost incurred by the retailer during the first cycle is expressed as:

$$OC_{r1} = c_{3r} \quad (4.16)$$

10. The ordering cost for the retailer over n complete cycles is given by:

$$OC_r = \sum_{j=1}^{n} OC_{r1}e^{-r(j-1)T} = c_{5r}\left\{\frac{1-e^{-rnT}}{1-e^{-rT}}\right\} \quad (4.17)$$

11. The purchasing cost incurred by the retailer during the first cycle is calculated as:

$$PC_{r1} = c_{6r}\left[a\left(t_1+\frac{bt_1^{\,2}}{2}+\frac{\theta t_1^{\,3}}{6}\right)+\frac{ae^{-(r+\eta)T}}{\eta}\left(e^{\eta T}-e^{\eta t_1}\right)\right] \quad (4.18)$$

12. The purchasing cost for the retailer across n complete cycles is expressed as:

$$PC_r = \sum_{j=1}^{n} PC_{r1}e^{-r(j-1)T} = c_{6r}\left[a\left(t_1+\frac{bt_1^{\,2}}{2}+\frac{\theta t_1^{\,3}}{6}\right)+\frac{ae^{-(r+\eta)T}}{\eta}\left(e^{\eta T}-e^{\eta t_1}\right)\right]\left\{\frac{1-e^{-rnT}}{1-e^{-rT}}\right\} \quad (4.19)$$

The retailer's total cost is obtained by summing the holding cost, deterioration cost, backordering cost, lost sales cost, ordering cost, and purchasing cost. Therefore, the total cost for n complete retailer cycles (considering $t_1=\gamma T$ with $0<\gamma<1$) is given by:

$$TC_r(n,T) = HC_r + DC_r + BC_r + LC_r + OC_r + PC_r \quad (4.20)$$

Case II Inventory model to design for the Supplier

The supplier initiates production at time t=0, and the inventory level gradually increases until $t=T_1$, the point at which production ceases. After production stops, the supplier's inventory declines from $t=T_1$ to $t=T_2$ due to the combined effects of demand fulfillment and product deterioration, eventually reaching zero. Throughout the interval $[0,T_2]$, the supplier dispatches n shipment batches to the retailer. The supplier's demand rate d is defined such that the order size satisfies q=dT. The variation of supplier inventory over the cycle is illustrated in Figure 2 and is governed by the following differential equations:

$$I_s'(t) = P - d - \theta t I_s(t), \qquad 0 \le t \le T_1 \quad (4.21)$$

$$I_s'(t) = -d - \theta t I_s(t), \qquad T_1 \le t \le T_2 \quad (4.22)$$

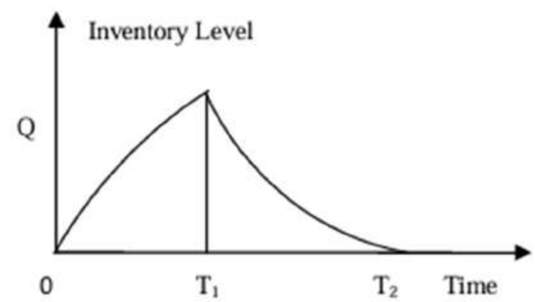

Figure 2: Forward manufacturing inventory configuration for the supplier

Solving the above differential equation under the boundary conditions $I_s(0)=0$ and $I_s(T_2)=0$, we obtain:

$$I_s(t) = (P-d)\left(t + \frac{\theta t^3}{6}\right)e^{-\theta t^2/2}, \quad 0 \le t \le T_1 \quad (4.23)$$

$$I_s(t) = d\left[\left(T_2 + \frac{\theta T_2^3}{6}\right) - \left(t + \frac{\theta t^3}{6}\right)\right]e^{-\theta t^2/2}, \qquad T_1 \le t \le T_2 \quad (4.24)$$

At time $t=T_1$, the inventory level remains continuous; therefore, we obtain:

$$T_2 = \frac{-4(2)^{1/3} d^2\theta + (2)^{2/3}\left[6d^2P\theta^2T_1 + d^2P\theta^3T_1^3 + \sqrt{d^4\theta^3\left\{32d^2 + P^2\theta T_1^2\left(6+\theta T_1^2\right)^2\right\}}\right]^{2/3}}{2d\theta\left[6d^2P\theta^2T_1 + d^2P\theta^3T_1^3 + \sqrt{d^4\theta^3\left\{32d^2 + P^2\theta T_1^2\left(6+\theta T_1^2\right)^2\right\}}\right]^{1/3}} \tag{4.25}$$

3 Forward and Reverse Manufacturing for the Supplier

This section is structured into two subsections.

Forward Manufacturing Model for the Supplier

The supplier's manufacturing process involves several cost components.

1. **Inventory holding cost for the supplier is expressed as:**

$$HC_s = c_{1s}\left[\int_0^{T_1} I_s(t)e^{-rt}dt + \int_{T_1}^{T_2} I_s(t)e^{-rt}dt\right]$$

$$HC_s = c_{1s}\left[\frac{P-d}{r^6}\left(r^4 - 2r^2\theta - 10\theta^2\right) - \frac{de^{-rT_1}}{r^3}\left(T_2 + \frac{\theta T_2^3}{6}\right)\left(\frac{\theta r^2T_1^2}{2} + \theta rT_1 + \theta - r^2\right)\right.$$

$$+\frac{Pe^{-rT_1}}{r^6}\left\{\frac{\theta r^5T_1^3}{3} - r^5T_1 - r^4 + \theta r^4T_1^2 + 2\theta r^3T_1 + 2\theta r^2 + \frac{\theta^2r^5T_1^5}{12} + \frac{5\theta^2r^4T_1^4}{12} + \frac{5\theta^2r^3T_1^3}{3}\right.$$

$$\left.\left.+5\theta^2r^2T_1^2 + 10\theta^2rT_1 + 10\theta^2\right\} - \frac{de^{-rT_2}}{r^6}\left\{\theta r^3T_2 - r^4 + 2\theta r^2 + \frac{\theta^2r^4T_2^4}{4} + \frac{3\theta^2r^3T_2^3}{2} + 5\theta^2r^2T_2^2 + 10\theta^2rT_2 + 10\theta^2\right\}\right] \tag{4.26}$$

2. The deterioration cost formulated for the supplier is given by:

$$DC_s = c_{2s}\left[\int_0^{T_1} \theta tI_s(t)e^{-rt}dt + \int_{T_1}^{T_2} \theta tI_s(t)e^{-rt}dt\right]$$

$$DC_s = c_{2s}\theta\left[\frac{P-d}{r^7}\left(2r^4 - 8r^2\theta - 60\theta^2\right) - \frac{de^{-rT_1}}{r^4}\left(T_2 + \frac{\theta T_2^3}{6}\right)\left(\frac{\theta r^3T_1^3}{2} + \frac{3\theta r^2T_1^2}{2} + 3\theta rT_1 + 3\theta - r^3T_1 - r^2\right)\right.$$

$$+\frac{Pe^{-rT_1}}{r^7}\left\{\frac{\theta r^6T_1^4}{3} - r^6T^2{}_1 - 2r^5T_1 - 2r^4 + \frac{4\theta r^5T_1^3}{3} + 4\theta r^4T_1^2 + 8\theta r^3T_1 + 8\theta r^2 + \frac{\theta^2r^6T_1^6}{12} + \frac{\theta^2r^5T_1^5}{2}\right.$$

$$\left.+\frac{5\theta^2r^4T_1^4}{2} + 10\theta^2r^3T_1^3 + 30\theta^2r^2T_1^2 + 60\theta^2rT_1 + 60\theta^2\right\} - \frac{de^{-rT_2}}{r^7}\left\{-r^5T_2 - 2r^4 - \frac{\theta^2r^5T_2^5}{6} + \theta r^4T_2^2\right.$$

$$\left.\left.+2r^4\theta^2T_2^4 + 5\theta r^3T_2 + \frac{19\theta^2r^3T_2^3}{2} + 8\theta r^2 + 30\theta^2r^2T_2^2 + 60\theta^2rT_2 + 60\theta^2\right\}\right] \tag{4.27}$$

3. The setup cost formulated for the supplier is expressed as:

$$SEC_s = c_{3s} \tag{4.28}$$

4. The production cost formulated for the supplier is given as:

$$PC_s = c\int_0^{T_1} Pe^{-rt}\,dt = \frac{cP}{r}\left(1-e^{-rT_1}\right) \tag{4.29}$$

Reverse Manufacturing Designed for the Supplier

The buyer orders the product but instead of receiving the shipment at once, they receive finished products periodically. After using the products, the supplier takes back those products. The reverse supply chain starts with the collecting of end products from customers, and proceeds with cleaning and disassembling processes. Following this collecting, sorting and dismantling service, some of the collected equipment is appropriate for refurbishment and restoration. It is believed that the remanufactured units would continue to satisfy purchasers and meet high quality criteria. Fig. shows the entire reverse manufacturing process. 3 and is controlled by the differential equations listed below.

$$I'_{sc}(t) = \delta a - \theta t I_{sc1}(t), \qquad 0 \le t \le T_1 \tag{4.30}$$

$$I'_{sc}(t) = -(R-\delta a) - \theta t I_{sc2}(t), \quad T_1 \le t \le T_2 \tag{4.31}$$

$$I'_{sR}(t) = (1-r_{sm})R - \theta t I_{sR1}(t), \quad T_1 \le t \le T_2 \tag{4.32}$$

$$I'_{sR}(t) = -d - \theta t I_{sR2}(t), \qquad T_2 \le t \le T_3 \tag{4.33}$$

Using the boundary conditions $I_{sc1}(0)=0, I_{sc2}(T_2)=0, I_{sR}(T_2)=0 \, and \, I_{sR}(T_3)=0$ we have

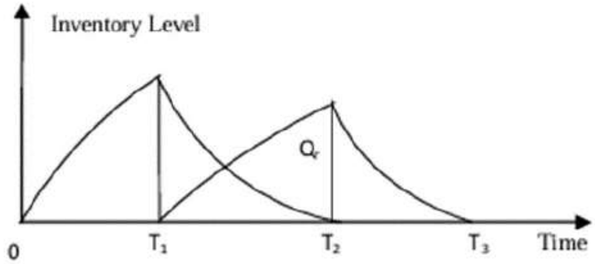

Figure 3: Inventory representation for the supplier under reverse manufacturing

$$I_{sc}(t) = \delta a\left(t + \frac{\theta t^3}{6}\right)e^{-\theta t^2/2}, \qquad 0 \le t \le T_1 \tag{4.34}$$

$$I_{sc}(t) = \left(R - \delta a\right)\left[\left(T_2 + \frac{\theta T_2^3}{6}\right) - \left(t + \frac{\theta t^3}{6}\right)\right]e^{-\theta t^2/2}, \quad T_1 \le t \le T_2 \tag{4.35}$$

$$I_{sR1}(t) = (1 - r_{sm})R\left[\left(t + \frac{\theta t^3}{6}\right) - \left(T_1 + \frac{\theta T_1^3}{6}\right) - \right]e^{-\theta t^2/2}, \qquad T_1 \le t \le T_2 \tag{4.36}$$

$$I_{sR2}(t) = d\left[\left(T_3 + \frac{\theta T_3^3}{6}\right) - \left(t + \frac{\theta t^3}{6}\right)\right]e^{-\theta t^2/2}, \quad T_2 \le t \le T_3 \tag{4.37}$$

By applying the continuity condition of the inventory level at times $t=T_1$ and $t=T_2$, we obtain:

$$R = \frac{\delta a\left(T_2 + \frac{\theta T_2^3}{6}\right)}{\left[\left(T_2 + \frac{\theta T_2^3}{6}\right) - \left(T_1 + \frac{\theta T_1^3}{6}\right)\right]} \tag{4.38}$$

$$T_3 = \frac{4(2)^{1/3} d^2\theta - (2)^{2/3}\left[\begin{array}{l}(1-r_{sm})\left\{d^2R\theta^2T_1\left(6+\theta T_1^2\right) - d^2R\theta^2T_2\left(6+\theta T_2^2\right)\right\} - d^3\theta^2T_2\left(6+\theta T_2^2\right)\\ +\sqrt{d^4\theta^3\left\{32d^2 + \theta\left\{(r_{sm}-1)\left(RT_1\left(6+\theta T_1^2\right) - RT_2\left(6+\theta T_2^2\right)\right) + dT_2\left(6+\theta T_2^2\right)\right\}^2\right\}}\end{array}\right]^{2/3}}{2d\theta\left[\begin{array}{l}(1-r_{sm})\left\{d^2R\theta^2T_1\left(6+\theta T_1^2\right) - d^2R\theta^2T_2\left(6+\theta T_2^2\right)\right\} - d^3\theta^2T_2\left(6+\theta T_2^2\right)\\ +\sqrt{d^4\theta^3\left\{32d^2 + \theta\left\{(r_{sm}-1)\left(RT_1\left(6+\theta T_1^2\right) - RT_2\left(6+\theta T_2^2\right)\right) + dT_2\left(6+\theta T_2^2\right)\right\}^2\right\}}\end{array}\right]^{1/3}} \tag{4.39}$$

1. The holding cost associated with the remanufacturing process, as formulated for the supplier, is given by:

$$HC_{sR}=\left[c_{1sC}\left\{\int_0^{T_1} I_{sc}(t)e^{-rt}dt+\int_{T_1}^{T_2} I_{sc}(t)e^{-rt}dt\right\}+c_{1sR}\left\{\int_{T_1}^{T_2} I_{sR}(t)e^{-rt}dt+\int_{T_2}^{T_3} I_{sR}(t)e^{-rt}dt\right\}\right]$$

$$HC_{sR}=\left[c_{1sC}\left[\frac{\delta a}{r^6}\left(r^4-2\theta r^2-10\theta^2\right)+\frac{(R-\delta a)e^{-rT_2}}{r^6}\left\{r^4-\frac{\theta^2 r^4T_2^4}{4}-\theta r^3T_2-\frac{3\theta^2 r^3T_2^3}{2}-2\theta r^2\right.\right.\right.$$

$$\left.-5\theta^2r^2T_2^2-10\theta^2rT_2-10\theta^2\right\}-\frac{(R-\delta a)e^{-rT_1}}{r^3}\left(T_2+\frac{\theta T_2^3}{6}\right)\left(\frac{\theta r^2T_1^2}{2}+\theta rT_1+\theta-r\right)$$

$$+\frac{Re^{-rT_1}}{r^6}\left\{-r^5T_1-r^4+\frac{\theta r^5T_1^3}{3}+\theta r^4T_1^2+2\theta r^3T_1+2\theta r^2+\frac{\theta^2r^5T_1^5}{12}+\frac{5\theta^2r^4T_1^4}{12}+\frac{5\theta^2r^3T_1^3}{3}\right.$$

$$\left.\left.+5\theta^2r^2T_1^2+10\theta^2rT_1+10\theta^2\right\}\right]+c_{1sR}\left[\frac{\left\{(1-r_{sm})R+d\right\}e^{-rT_2}}{r^6}\left\{-r^5T_2-r^4+\frac{\theta r^5T_2^3}{3}+\theta r^4T_2^2\right.\right.$$

$$\left.+2\theta r^3T_2+2\theta r^2+\frac{\theta^2r^5T_2^5}{12}+\frac{5\theta^2r^4T_2^4}{12}+\frac{5\theta^2r^3T_1^3}{3}+5\theta^2r^2T_1^2+10\theta^2rT_1+10\theta^2\right\}$$

$$+\frac{(1-r_{sm})Re^{-rT_1}}{r^6}\left\{r^4-\theta r^3T_1-2\theta r^2-\frac{5\theta^2r^4T_1^4}{12}-\frac{3\theta^2r^3T_1^3}{2}+\theta^2r^4T_1^5-5\theta^2r^2T_1^2-10\theta^2rT_1-10\theta^2\right\}$$

$$+\frac{de^{-rT_3}}{r^6}\left\{r^4-\theta r^3T_3-2\theta r^2-\frac{5\theta^2r^4T_3^4}{12}-\frac{3\theta^2r^3T_3^3}{2}+\theta^2r^4T_3^5-5\theta^2r^2T_3^2-10\theta^2rT_3-10\theta^2\right\}$$

$$\left.\left.-\frac{e^{-rT_2}}{r^3}\left(\theta-r^2+\frac{\theta r^2T_2^2}{2}+\theta rT_2\right)\left\{(1-r_{sm})R\left(T_1+\frac{\theta T_1^3}{6}\right)+d\left(T_3+\frac{\theta T_3^3}{6}\right)\right\}\right]\right] \tag{4.40}$$

2. The supplier’s deterioration cost associated with remanufactured products is expressed as:

$$DC_{sR}=\left[c_{2sC}\left\{\int_0^{T_1}\theta tI_{sc}(t)e^{-rt}dt+\int_{T_1}^{T_2}\theta tI_{sc}(t)e^{-rt}dt\right\}+c_{2sR}\left\{\int_{T_1}^{T_2}\theta tI_{sR}(t)e^{-rt}dt+\int_{T_2}^{T_3}\theta tI_{sR}(t)e^{-rt}dt\right\}\right]$$

$$
\begin{aligned}
DC_{sR} = \Bigg[& c_{2sC}\theta\Bigg[\frac{\delta a}{r^7}\left(2r^4-8\theta r^2-60\theta^2\right)-\frac{(R-\delta a)}{r^4}\left(T_2+\frac{\theta T_2^3}{6}\right)\Bigg(-r^3T_1-r^2+\frac{\theta r^3T_1^3}{2}+\frac{3\theta r^2T_1^2}{2} \\
& +3\theta rT_1+3\theta\Bigg)e^{-rT_1}+\frac{Re^{-rT_1}}{r^7}\Bigg(-r^6T_1^2-2r^5T_1-2r^4+\frac{\theta r^6T_1^4}{3}+\frac{4\theta r^5T_1^3}{3}+4\theta r^4T_1^2+8\theta r^3T_1+8\theta r^2 \\
& +\frac{\theta^2r^6T_1^6}{12}+\frac{\theta^2r^5T_1^5}{2}+\frac{5\theta^2r^4T_1^4}{2}+10\theta^2r^3T_1^3+30\theta^2r^2T_1^2+60\theta^2rT_1+60\theta^2\Bigg)+\frac{(R-\delta a)e^{-rT_2}}{r^7} \\
& \left(r^5T_2-\theta r^4T_2^2-5\theta r^3T_2-\frac{\theta^2r^5T_2^5}{4}-2\theta^2r^4T_2^4-\frac{19\theta^2r^3T_2^3}{2}+2r^4-8r^2\theta-30\theta^2r^2T_2^2-60\theta^2rT_2-60\theta^2\right)\Bigg] \\
& +c_{2sR}\theta\Bigg[\frac{\{(1-r_{sm})R+d\}e^{-rT_2}}{r^7}\Bigg(-r^6T_2^2-2r^5T_2-2r^4+\frac{\theta r^6T_2^4}{3}+\frac{4\theta r^5T_2^3}{3}+4\theta r^4T_2^2+8\theta r^3T_2+8\theta r^2 \\
& +\frac{\theta^2r^6T_2^6}{12}+\frac{\theta^2r^5T_2^5}{2}+\frac{5\theta^2r^4T_2^4}{2}+10\theta^2r^3T_2^3+30\theta^2r^2T_2^2+60\theta^2rT_2+60\theta^2\Bigg)+\frac{(1-r_{sm})Re^{-rT_1}}{r^7}\Big(r^5T_1 \\
& -\theta r^4T_1^2-5\theta r^3T_1-\frac{\theta^2r^5T_1^5}{4}-2\theta^2r^4T_1^4-\frac{19\theta^2r^3T_1^3}{2}+2r^4-8r^2\theta-30\theta^2r^2T_1^2-60\theta^2rT_1-60\theta^2\Big)+\frac{de^{-rT_3}}{r^7} \\
& \left(r^5T_3-\theta r^4T_3^2-5\theta r^3T_3-\frac{\theta^2r^5T_3^5}{4}-2\theta^2r^4T_3^4-\frac{19\theta^2r^3T_3^3}{2}+2r^4-8r^2\theta-30\theta^2r^2T_3^2-60\theta^2rT-60\theta^2_3\right) \\
& -\frac{e^{-rT_2}}{r^4}\left\{(1-r_{sm})R\left(T_1+\frac{\theta T_1^3}{6}\right)+d\left(T_3+\frac{\theta T_3^3}{6}\right)\right\}\left\{-r^3T_2-r^2+\frac{\theta r^3T_2^3}{2}+\frac{3\theta r^2T_2^2}{2}+3\theta rT_2+3\theta\right\}\Bigg]\Bigg]
\end{aligned} \tag{4.41}
$$

3. The cost incurred for cleaning and dismantling activities within the supplier's remanufacturing setup is represented as:

$$CDC_{sR}=F_{cL}+c_{cL}\int_0^{T_2}\delta ae^{-rt}dt=F_{cL}+\frac{c_{cL}\delta a}{r}\left(1-e^{-rT_2}\right) \tag{4.42}$$

4. The cost incurred by the supplier for carrying out the remanufacturing process is given by:

$$RMC_{sR}=\frac{F_{rm}}{L}+c_{rm}\left(1-e^{-\lambda_{sm}T_2}\right)\int_{T_1}^{T_2}L(1-r_{sm})r_{rm}Re^{-rt}dt$$

$$=\frac{F_{rm}}{L}+\frac{c_{rm}\left(1-e^{-\lambda_{sm}T_2}\right)L(1-r_{sm})r_{rm}R}{r}\left(e^{-rT_1}-e^{-rT_2}\right) \tag{4.43}$$

5. The cost associated with converting recovered products into reusable items, as formulated for the supplier, is expressed as:

$$CVC_{sR} = \frac{F_{cv}}{L} + c_{cv}\left(1-e^{-\lambda_c T_2}\right)\int_{T_1}^{T_2} L(1-r_{sm})Re^{-rt}\,dt$$

$$= \frac{F_{cv}}{L} + \frac{c_{cv}\left(1-e^{-\lambda_c T_2}\right)L(1-r_{sm})R}{r}\left(e^{-rT_1}-e^{-rT_2}\right) \tag{4.44}$$

6. The repair cost considered for the supplier incorporates the functional life of components, assuming that the hazard rate follows a Weibull distribution with LT_2 as the designed service life and β as the shape parameter. Accordingly, the resulting expression for the failure rate is:

$$F_R = \int_0^{LT_2} \alpha^{\beta}\beta t^{\beta-1}\,dt = (\alpha LT_2)^{\beta} \tag{4.45}$$

7. The supplier's total repair expense, based on the established model, is given by:

$$RC_{sR} = \frac{F_{rp}}{L} + c_{rp}\int_{T_1}^{T_2} F_R R(1-r_{sm})e^{-rt}\,dt$$

$$= \frac{F_{rp}}{L} + \frac{c_{rp}(\alpha LT_2)^{\beta}(1-r_{sm})R}{r}\left(e^{-rT_1}-e^{-rT_2}\right) \tag{4.46}$$

8. The salvage cost assigned to the supplier decreases as a result of design improvements that enhance functionality. Therefore, after the processes of cleaning, disassembly, sorting, and identification in the reverse logistics stream, the salvage cost for the supplier is expressed as:

$$SLC_{sR} = c_{sL}\int_{T_1}^{T_2} r_{sm}R(1-k_s b_{Lc})e^{-rt}\,dt = \frac{c_{sL}(1-k_s b_{Lc})r_{sm}R}{r}\left(e^{-rT_1}-e^{-rT_2}\right) \tag{4.47}$$

4 Designing the Product Life-Cycle through Functional Up-Gradation

The supplier's product life-cycle design cost (L) is composed of the expenses related to green design along with the cost of enhancing product functionality. Thus, the present value of the overall life-cycle design cost is expressed as:

$$PLC_{sR} = c_{Ld}\left\{\frac{a_{Lc}}{L} + \frac{b_{Lc}}{r}(1-e^{-rL})\prod_{i=1}^{k}R_i\right\} + c_{Lu}\left\{\frac{a_{Lu}}{L} + \frac{b_{Lu}}{r}(1-e^{-rL})\prod_{i=1}^{k}R_i\right\} \tag{4.48}$$

The supplier's cost arising from limited flexibility in carrying out just-in-time deliveries for a total of n shipments is expressed as:

$$LFC_s = \sum_{j=1}^{n} c_{LF} e^{-r(j-1)T} = c_{LF} \left\{ \frac{1-e^{-rnT}}{1-e^{-rT}} \right\} \quad (4.49)$$

Assuming that each delivery is paired with one assessment, and that the assessment occurs at the start of every cycle, the total assessment cost for n deliveries, as designed for the supplier, is expressed as:

$$INC_s = \sum_{j=1}^{n} c_{iN} e^{-r(j-1)T} = c_{iN} \left\{ \frac{1-e^{-rnT}}{1-e^{-rT}} \right\} \quad (4.50)$$

The total cost incurred by the supplier throughout the forward production phase and the reverse manufacturing process can be expressed as:

$$\begin{aligned} TC_s = {} & PC_s + HC_s + DC_s + SEC_s + PC_{sR} + HC_{sR} + DC_{sR} + CDC_{sR} \\ & + RMC_{sR} + CVC_{sR} + RC_{sR} + SLC_{sR} + PLC_{sR} + LFC_s + INC_s \end{aligned} \quad (4.51)$$

The overall expenditure associated with the entire supply chain is given by:

$$TC(\mathrm{n}, \mathrm{T}, T_1) = TC_r + TC_s \quad (4.52)$$

It is observed that the total cost depends on the variables n, t_1, T, and T_1 through equations (4.25), (4.38), and (4.39). Since "n" is a discrete decision variable, the optimal value of n is determined by fulfilling the condition: $TC((n-1), T^*, T_1^*) \geq TC(n, T^*, T_1^*) \leq TC((n+1), T^*, T_1^*)$ where T^* and T_1^* represent the optimal costs corresponding to T and T_1. Therefore, the remaining task is to determine the optimal values of n, T, and T_1 such that:

$$\begin{aligned} & \text{Minimize } \mathrm{TC}(\mathrm{n}, \mathrm{T}, T_1) \\ & \textit{Subject to}: TC((n-1), T^*, T_1^*) \geq TC(n, T^*, T_1^*) \leq TC((n+1), T^*, T_1^*) \\ & \qquad T > 0, T_1 > 0 \end{aligned} \quad (4.53)$$

5. Numerical Example

In this section, we investigate the processing of decaying items in a two-echelon supply chain which consists of single dealer and single supplier by incorporating recycling and recovery. In order to provide a realistic numerical example, a full set of cost parameters and operational details are considered here. The holding cost H for the retailer taken as 0.8 Rs per unit and deterioration C R is considered as 0.9 Rs per unit. The backordering cost of shortfall is 10 Rs per unit and the lost sales are 20 Rs per unit. Further, the ordering cost per cycle for any replenishment order set by the retailer is 90 Rs and the acquiring cost of new items from supplier is 15 Rs per unit. A rate coefficient (interactional speed) that considers the effects of 0.06. These parameters describe the economic condition in which the retailer plays.

Multiple specific cost elements and operational variables are introduced for supplier in both forward manufacturing phase and reverse operation period. The holding cost of fresh products is assumed as Rs.0.89 perunit and the deterioration cost for new inventory is assumed as 0.9 Rs/1 unit. The set up cost of starting a new production cycle is supposed to be 150 Rs per cycle. The production rate during the forward manufacturing is assumed to be 3 units per time unit. For the type-2 demand, if collected used items are stored from customers, supplier has to bear a holding cost (h) of 0.07 Rs per item and deterioration cost = 0.08 Rs per unit for these stored used items which are reused in processing again. The holding cost for the reused items at the supplier site will be 0.8 Rs per unit (product), similarly, deterioration cost for remanufacturing inventory stage is assumed to be 0.8 Rs per unit. The cleaning and disassambly cost of the items' returned for each cycle is assumed to be 150 Rs. The remanufacturing, convertibility and renovation process related costs are fixed at 150 Rs per cycle in each case. For every supplier's delivery an extra cost of 120 Rs due to low flexibility is involved. The examination charge for each delivery is also considered as 150 Rs.

The life- cycle features of the product also affect the reverse manufacturing and recycling systems. The number of life cycles is assumed to be three before an item will ultimately be thrown away. In an environmental and sustainability context, product life-cycle design costs have been considered. Costs for green-design (5 RS per unit,) and for upgrading are 5 RS per unit. Extra cleaning and dismantling charges for recycling will be considered as 0.5 Rs per unit. The reman process is characterized by the following variable costs 2 Rs per unit for remain, 2 Rs per unit for convertibility and 2 Rs per unit for repair. After the cleaning, sorting and disassembly activities, unworkable parts yield a recovery cost of 6 Rs per piece. Second, 0.5 is assumed for the percentage of return items remanufactured after cleaning and disassembly, and same 0.5 for the amount of refurbished in actual stage of remanufacturing. The entering rate of used item back to the system is 0:02 units and there are 2 % adaptability factor. The arrival rate of items requiring repair is $\alpha = 0.001$ units. The failure behavior of components follows a Weibull distribution with shape parameter $\beta = 1$. The proportionality constant ks is taken as 0.5. In addition, the supplier incurs product life-cycle design costs for resource reuse and green design, including a fixed cost of 6 units and a variable cost of 0.6 units. Similarly, for upgrading the design of products across life cycles, the fixed cost is taken as 2 units, and the variable cost is 0.4 units. The reliability values associated with the supplier's sub-functions are assumed to be $R_1 = 0.99$ and $R_2 = 0.98$.

On the demand side, the study assumes a stock-dependent demand pattern at the retailer, characterized by the parameters a = 45 and b = 0.02. The fraction of the cycle before shortages begin is $\gamma = 0.75$. The backlogging parameter governing customers' willingness to wait is taken as $\eta = 0.4$. The deterioration rate of items is $\theta = 0.01$. The resale or assembly value of second-hand items is taken as 0.7 Rs. For the supplier, the production rate in the forward system is assumed to be 100 units. With these decisions, the decision maker plans to find several important performance measures: the optimal number of deliveries from supplier shipping center to retailer's demand site, the cycle length of retailer and its production period for new items at supplier for both time periods as well as remanufacturing rate in reverse process by also glorious total cost in supply chain system. Those optimal values can be determined by the solution approach for the presented inventory model. Numerical solutions are obtained with the software Mathematica 10.0. The ultimate best outputs are given in Table 1, which yields practical guidelines for the supply chain decision making and efficiency enhancement.

Table 1: Best possible consequences for integrated, cost minimization policy

n	T	T_1	T_2	T_3	D	R	TC
4	1.79815	2.45227	3.31692	4.11429	79.1022	150.734	18684.4

Based on Table 1, when the supplier makes four deliveries to the retailer, the retailer's cycle time is 1.7985. The supplier's production rate for new items is 2.45227, while the inventory of fresh items at the supplier reaches zero at time 3.31692. The supplier's remanufactured inventory depletes at time 4.11429. The supplier's demand is 79.1022, and the remanufacturing rate for returned items is 150.734. Under these conditions, the total supply chain cost amounts to 1864.4.

6. Sensitivity Analysis

We now examine how variations in the system parameters c_{1r}, c_{2r}, the demand parameter "b", and the production cost parameter "c" influence the model.

Table 2: Sensitivity analysis for the parameter c_{1r}

c_{1r}	T	T_1	T_2	T_3	N	d	R	TC
0.75	1.81081	3.42907	4.29633	5.09193	4	78.9493	149.639	19448.4
0.78	1.80320	3.44297	4.30867	5.10533	4	79.0411	150.294	19583.4
0.82	1.79312	3.46161	4.32523	5.12332	4	79.1631	151.174	19766.4
0.85	1.78558	3.47570	4.33775	5.13692	4	79.2547	151.841	19906.1

Sensitivity analysis shown in Table 2 indicates that gradual increases of the parameter cr1 result in systematic alternations of performance measures to all parts of the model. While the values of T, T1, T2, and T3 show a slight decreasing then adjusting trend as $c_{ır}$ goes from 0.75 to 0.85 suggesting intermediate sensitivity of cycle-related times with respect to cost parameters. Furthermore, d, R and TC are increasing functions of c_{1r} implies that large c_{Ir} raise demand rate, replenishment rate and total cost. In general, Tables 2 shows a positive dependent relationship between $c_{ır}$ and system cost behavior.

Table 3: Sensitivity analysis for the parameter c2r

c_{2r}	T	T_1	T_2	T_3	N	d	R	TC
0.80	1.93267	3.22389	4.11472	4.89495	4	77.5200	140.124	17687.6
0.85	1.86971	3.32598	4.20495	4.99275	4	78.2491	144.822	18532.6
0.88	1.82849	3.39729	4.26813	5.06131	4	78.7373	148.146	19163.9
0.89	1.81371	3.42380	4.29165	5.08685	4	78.9144	149.391	19407.5

According to Table 3 it can be noticed that for varying c_2r, significant changes in the operational performance of the model understood well together. Rising of c_2r from 0.80 to 0.89, T, T1, T2 and T3 gradually decrease and then have tiny variation, indicating that cycle-time components are moderately sensitive to this factor. Concurrently, d, R and TC increase monotonously,® as can be seen that higher c_2 raises the demand, the replenishment rate and the overall cost. Overall, Table 3 verifies a positive relationship between c_2r and system cost behavior.

Table 4: Sensitivity analysis for the demand parameter b

b	T	T_1	T_2	T_3	N	d	R	TC
0.015	1.79254	3.44535	4.31078	5.10762	4	79.0567	150.407	19590.7
0.019	1.79703	3.45086	4.31567	5.11293	4	79.0930	150.667	19657.4
0.021	1.79927	3.45368	4.31818	5.11566	4	79.1114	150.800	19691.5
0.025	1.80372	3.45944	4.32330	5.12123	4	79.1489	151.072	19761.0

The sensitivity analysis presented in Table 4 reveals that increasing demand parameter b can result in continuous and well predictable adaptations of system behavior. While b increases from 0.015 to 0.025, T, T1, T2 and T3 also increase slightly that reflects the weak effect of demand intensity upon cycle-time components as well. The d, R and TC all move up slowly with the demand, indicating the fact that high demand will make replenishing volume requirement and total system cost increase correspondingly. In general, Table 4 supports a positive association between b and measures of operating costs.

Table 5: Sensitivity analysis for the production cost parameter c

c	T	T_1	T_2	T_3	N	d	R	TC
2.0	1.79803	3.45262	4.31728	5.11469	4	79.1036	150.744	19365.9
2.5	1.79809	3.45244	4.31710	5.11449	4	79.1029	150.738	19520.1
3.5	1.79821	3.45209	4.31675	5.11410	4	79.1014	150.727	19828.6
4.0	1.79828	3.45192	4.31658	5.11391	4	79.1006	150.722	19982.9

The results in Table 5 show that the system-cycle-time components are not very sensitive to changes in the production cost parameter c. With the increase of c from 2.0 to 4.0, the values of T, T1, T2 and T3 were fairly constant indicating very low sensitivity timing measures with production cost fluctuations. On the other hand, TC increases monotonically with, which indicates that production cost directly adds to total system cost. In contrast d and R stay almost the same. Taken all in all, Table 5 reveals that c mainly influences total cost rather than operational timings.

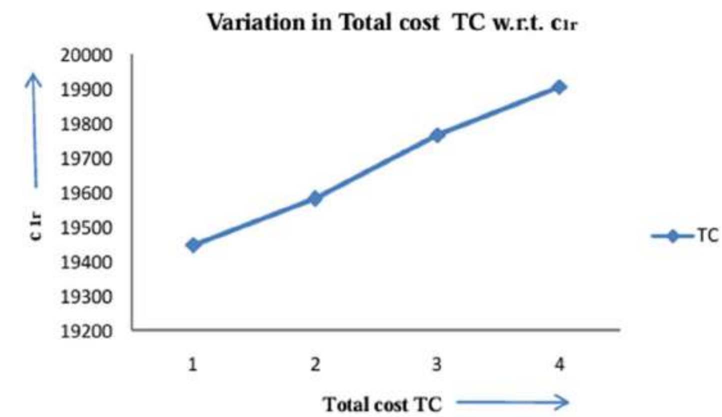

Figure 4: Illustration of Total Cost (TC) in Relation to c_{1r}

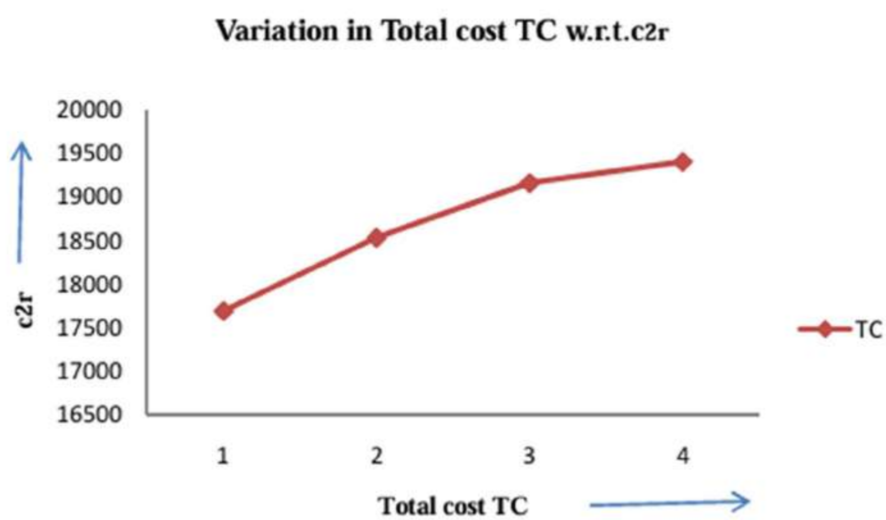

Figure 5: Illustration of Total Cost (TC) with Respect to c_{2r}

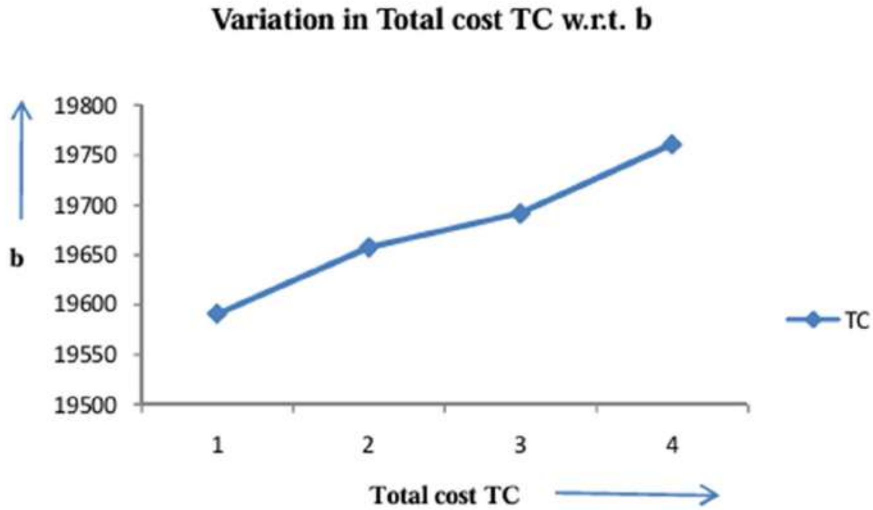

Fig. 6: Illustration of Total Cost (TC) in Relation to the Demand Parameter b

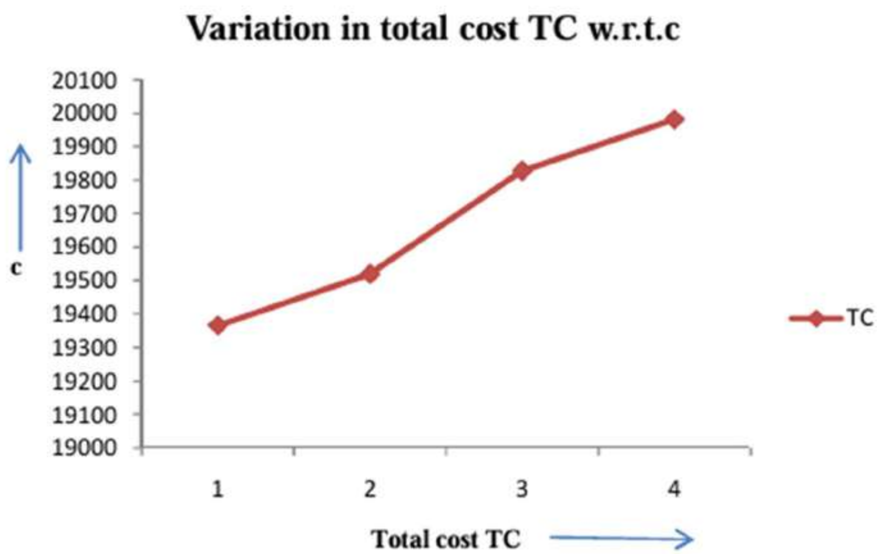

Figure 7: Illustration of the Total Cost (TC) with Respect to the Production Cost Parameter c

7. Observations

Through Tables 1- 5, and Figures 4 - 7 the behavior of the integrated cost minimization model is analyzed across different system parameters. The optimal overall results for the integrated policy are presented in Table 1, and show that the chosen numbers of deliveries and time-cycles solve the system at minimum costs. Moreover, by looking at the sensitivity analysis for parameter c1r given in Table 2 and Figure 4, the increase of c1r always increases total cost TC along with an increase of time-cycle variables T, T1, T2, T3, demand level d and remanufacturing rate R. Scenario-based results for parameter c4 are given in Table 5 as well as Figure 5. In addition we can infer from sensitivity results to Parameter c2r (shown on Table 4.3and illustrated by Figure 4.5) that when increasingc2r this will cause total cost TC among other time parametersT1, T2,T3,demanddand remanufacturing rate Rtoincrease which are inversely proportional to cycle length the latter decreases when growing c2r. The changes in demand parameter b appear in Table 4 and Fig. 6, which show higher total cost and other time variables T,T1,T2, T3 and the remanufacturing rate R when b raise. Finally, Table 4.5 and Figure 7 present how the increase of production cost parameter c affects individual costs. The results show that RMT project a higher total cost as well as longer T1 from increasing of c. T, T1, T2, T3. Conversely values of time parameters T, T2, T3, and demand d decrease with raising p. These observations generally serve to demonstrate the importance of cost- and demand-related factors in determining operational timings, remanufacturing rates, and total cost in the integrated supply chain model.

7. Conclusion

An integrated supply chain model for deteriorating products with rework of imperfect quality items under inflation has been constructed in this chapter. Although extant green supply chain and recycling models focus on improvement of sustainable supplier performance, environmental management strategies, green production and environmental friendly operations, there is generally no consideration of the impact of inflation and time-value-of-money on any recycling process. However, considering the cost associated with used products that need to be returned as much less as possible, in this paper we develop a joint retailer and supplier replenishment strategy which explicitly accounts for these economic factors. The chapter also examines how green product design, technological improvement and remanufacturing activities impact the manufacturing inventory decisions under partial shortages. It is shown that when the demand rate at the supplier grows up, not only holding costs of the retailer but also deterioration is enhanced and finally total supply chain cost rises. The sensitivity analysis for the system's four parameters c1r, c2r, b, and c is carried out to show how these values affect the system's performance and, as a result, provide some insight into the workings of our suggested model. The results indicate that the current model is more robust under industrial conditions than previous models. The viability of the theoretical findings is illustrated with a detailed numerical example, and a sensitivity analysis reveals that the model is not sensitive. For future work the further development would include adding other environmental decision tool and investigating various remanufacturing (or restoration) technologies to improve the sustainability of supply chain system.

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