ISSN 2277-2685
IJESR/Jan. 2016/ Vol-7/Issue-1/1-16
Padma Gahan *et. al.*, / International Journal of Engineering & Science Research

EMPIRICAL ANALYSIS OF NONLINEAR FINANCIAL MODELS

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ABSTRACT

A fractional-order derivative nonlinear financial model of economic dynamics is presented. By eliminating the limit operation, a derivative of the Jumari type may be discretized. The model's parameters and coefficients are estimated using the least squares method. An example application of this novel method for modeling financial systems is the modeling of the interest rate, investment, and inflation dynamics of the Indian national financial system. Empirical findings are shown over a range of discretization time steps, and the difference between the estimated data and the real data is displayed graphically. Interesting insights into the model are revealed by a comparison with a previous model of financial derivatives of integer order. The optimization of management strategy and decision technology in India's financial system may be greatly aided by using the fractional-order discrete approach to a nonlinear financial model. This reduces the risk of making incorrect predictions about the economy.

INTRODUCTION

Fractional calculus's roots may be traced back to the late seventeenth century, during the time when Newton and Leibniz were developing their theories of classical differential and integral calculus [13]. After the introduction of d for the first derivative in contemporary differential calculus, a later dated notation, dt, was developed.

In 1965, l'Hospital posed a question to Leibniz about d2, the derivative of order 1.
Derivatives and integrals of arbitrary real or complex order are the subject of fractional calculus, a term coined by the notation 1 2 2 dt 2. Therefore, the classical calculus that we are familiar with today is as ancient as the fractional calculus that is still in use today. The field of mathematics known as fractional calculus is 310 years old. Many studies have shown the usefulness of fractional models in describing physical and biological processes and systems. Examples of fields that use this are electro-analytical chemistry, neuron modeling, diffusion processes, damping laws, rheology, and visco-elasticity. Fractional derivatives have recently found use in the domains of psychology and biology [19]. Although the ideas and mathematics behind fractional differential equations date back many centuries, it was only within the last few decades that it was understood that these derivatives might be used in very effective modeling of the actual world. Despite its age, it was not used in the fields of physics, engineering, or finance for quite some time. However, during the last 20 years, not only mathematicians but also physicists, engineers, and financial analysts have shown an increasing interest in fractional calculus. Financial mathematics has also benefited from an expansion of fractality ideas [14]. The applications of fractional calculus to classical analysis are almost limitless.

Therefore, during the last 35 years, a growing number of researchers and analysts in the fields of physics, chemistry, engineering, life sciences, finance, and other disciplines have been drawn to the study of fractional differential equations. Numerical approaches are used to gain most conclusions concerning solving fractional differential equations since only a small subset of these equations can be solved analytically. Large-amplitude, periodic changes in the financial and economic system have garnered a lot of attention as of late [12, 15, 16, 17].

In light of this, it is challenging to statistically detect multicollinearity between various financial variables and economic factors when examining the dynamics of financial behavior. Several nonlinear financial models have been constructed in the last decade to investigate periodic, chaotic, and memory-based behaviors in financial and economic systems in light of the widespread recognition that these fields invariably exhibit nonlinearity. In particular, [2-6,9] examine the intricate dynamics of economic cycles by using the van der pol model. By introducing a forcing function, forced van der pol equations are useful for modeling the intricate interdependencies between economic variables. national economy and the international economy in an era of seasonal effects, such as the sun's cycle, on the ups and downs of the economy. Nonlinear dynamics of financial and economic systems have gained in popularity and precision thanks to these and other studies. All prior studies have focused on the nonlinear dynamics of financial and economic systems, namely on their cyclical and chaotic behaviors. A simplified macro-financial model of interest rates, investment demand, and inflation was built and evaluated [10, 11, 20] for things like equilibrium, periodic stability, chaotic behavior, and so on. By taking into account Goodwin's nonlinear accelerator model with periodic investment outlays, the complex motion in nonlinear dynamic systems is investigated in [8]. It is known that nonlinear economic models often display transient chaotic dynamics. Since chaotic dynamics has a negative impact on properly and successfully anticipating economic outcomes, there has been an uptick in studies aimed at analyzing and taming the nonlinear dynamics that underpin financial and economic markets.

By taking into account different amounts of time steps, this research finds the ideal fractional orders and parameters for discretizing financial systems. Unlike integer order models, fractional order models need access to the system's memory to function properly. Foreign exchange rates, GDP, interest rates, output, unemployment, and stock market prices are only few of the financial variables whose magnitudes may have extremely long memory, i.e. system history. It shows that the largest time scales in the financial markets coincide with the correlations [18]. What this implies is that past changes in financial variables may be used to predict future changes. In order to evaluate how well the empirical financial model is fitted, we create a new discrete financial model. This is the first continuous or discrete factional-order dynamic financial model of a country's or region's actual financial and economic data, with a focus on India, as far as the authors are aware. It is anticipated that the publication of this report would encourage more study in this area.
This work is structured as follows: Section 2 introduces the mathematical prerequisites. We describe the fractional form of the newly published integer-order financial model, namely the nonlinear dynamic econometric models of the financial system, and estimate the parameters using the concept of least squares in Section 3. In Section 4, we use discretized fractional-order optimization and estimate of a nonlinear financial model to conduct an empirical study of Indian economic and financial data from 1981 to 2015 and portray the dynamic behavior. Section 5 provides some last thoughts on the matter. The outcomes of current studies in this area are summarized in Table 1.

Table 1: Summary of Some Empirical Research on Financial Modelling

Author(s)	Period & Market		Characteristics	Principle	Comparative Analysis	Model
Chian et al. (2005)	-	Chian et al. Type	Order, Chaos, unstable periodic orbits, Chaotic saddles & Intermittency	Forced oscillator, saddle node bifurcation	No	Nonlinear Dynamic
Chen (2006)	-	Caputo-Type	Chaos	Chaos	No	Nonlinear Fractional-order
Xu et al. (2011)	-	Abel Differential Equation	Memory	Short Memory	No	Nonlinear Fractional-order
Yue et al. (2013)	1980-2011, Japan	Jumari Type	Dynamic behaviour	Least Square	No	Nonlinear Fractional-order
Present Study (2017)	1981-2014, India	Jumari Type	Dynamic behaviour	Least Square	Yes	Nonlinear Fractional-order

MATHEMATICAL PRELIMINARIES

Here, we lay the groundwork for understanding fractional derivatives.

For simplicity, we will refer to continuous functions as f(x): R R and constant dicretization spans as > 0. Here we provide a definition for the fractional difference of f(x) of order (R, 0 1).

$\Delta\alpha\ f(x) = \sum+\infty\ (-1)k\ (\alpha)\ f(x + (\alpha - k)\square)$ (1) k=0 k

Then, we may write D f(t) = lim f(t), where f is the fractional derivative of t.

□→0

$\Delta\alpha\ f(t)$

□ α

(2) For a continuously differentiable function u: [0,) R, the modified fractional derivative of Jumarie's order is defined as in [7].

$D\alpha\ f(t) = \Gamma(1 + \alpha - m)\lim \Delta\alpha\ fm\ (t)$ (3)

t □→0

□ α−m ,

where m m + 1, where m = [], and the integer component of the actual number is denoted by []. In addition, if 0 1,

$D\alpha\ f(t) = \Gamma(1 + \alpha)\lim \Delta\alpha\ f(t)$

(4)

t □→0

□ α

Several definitions of fractional derivative may be found in [1], however we will only be using Jumarie's definition here. The discrete version of Jumaie's fractional derivative may be expressed by the classical difference of function, multiplied by certain coefficients; this is possible because we can choose a short step size and eliminate the limit operation in (4). Using Jumaie's fractional derivative, we can now build both continuous and discrete financial models.

MODEL DESCRIPTION AND ESTIMATION METHODOLOGY

[11] have recently reported a dynamic model of finance, composed of three first order differential equations. The model describes the time-variation of three state variables: the interest rate, X, the investment demand, Y, and the price index, Z. The factors that influence changes in X mainly come from two aspects: first, contradictions from the investment market, i.e., the surplus between investment and savings, and second, structural adjustment from good prices. The changing rate of Y is in proportion to the rate of investment, and in proportion to an inversion with the cost of investment and interest rates. Changes in Z, on the one hand are controlled by a contradiction between supply and demand in commercial markets, and on the other hand, are influenced by inflation rates. By choosing an appropriate coordinate system and setting appropriate dimensions for every state variable, [11] offer the simplified finance model as:

$$\begin{aligned} \dot{X} &= Z + (Y - a)X, \\ \dot{Y} &= 1 - bY - X^2, \qquad (5) \\ \dot{Z} &= -X - cZ, \end{aligned}$$

where a is the saving amount, b is the cost per investment, and c is the elasticity of demand of commercial markets. It is obvious that all the three constants $a, b\ and\ c$ are nonnegative coefficients with economic interpretations.

Here, we consider the generalization of system (5) for the fractional incommensurate-order model which takes the form:

$$\begin{aligned} D_t^{q_1} X &= \frac{d^{q_1} X}{dt^{q_1}} = Z + (Y - a)X, \\ D_t^{q_2} Y &= \frac{d^{q_2} X}{dt^{q_2}} = 1 - bY - X^2, \qquad (6) \\ D_t^{q_3} Z &= \frac{d^{q_3} X}{dt^{q_3}} = -X - cZ \end{aligned}$$

where, $q_i \in (0,1]$ $(i = 1,2,3)$ represent the fractional order of the derivatives. If $q_1 = q_2 = q_3 = 1$, (6) reducesto the integer-order Chen system. Again the system (6) can written in the form of time variable t.

$$
\begin{aligned}
D_t^{q_1} x_t &= \frac{d^{q_1} x_t}{dt^{q_1}} = z_t + (y_t - a)x_t, \\
D_t^{q_2} y_t &= \frac{d^{q_2} y_t}{dt^{q_2}} = 1 - b y_t - x_t^2, \qquad (7) \\
D_t^{q_3} z_t &= \frac{d^{q_3} z_t}{dt^{q_3}} = -x_t - c z_t
\end{aligned}
$$

where, x, y *and* z represent the interest rate, investment, and inflation respectively. The subscript t indicates that the variable depends on t. Instead of considering the same expressions in fractional chaotic Chen system, we assume a more general form of the present financial model as:

$$
\begin{aligned}
D_t^{q_1} x_t &= c_1 + a_{11} x_t + a_{12} y_t + a_{13} z_t + a_{14} x_t^2 + a_{15} y_t^2 + a_{16} z_t^2 + a_{17} x_t y_t + a_{18} y_t z_t + a_{19} z_t x_t + u_{1t} = \\
&\text{The rate of change in investment with fractional order w.r.to time } t, \\
D_t^{q_2} y_t &= c_2 + a_{21} x_t + a_{22} y_t + a_{23} z_t + a_{24} x_t^2 + a_{25} y_t^2 + a_{26} z_t^2 + a_{27} x_t y_t + a_{28} y_t z_t + a_{29} z_t x_t + u_{2t} = \\
&\text{The rate of change in interest rate with fractional order w.r.to time } t, \qquad (8) \\
D_t^{q_3} z_t &= c_3 + a_{31} x_t + a_{32} y_t + a_{33} z_t + a_{34} x_t^2 + a_{35} y_t^2 + a_{36} z_t^2 + a_{37} x_t y_t + a_{38} y_t z_t + a_{39} z_t x_t + u_{3t} = \\
&\text{The rate of change in inflation with fractional order w.r.to time } t
\end{aligned}
$$

Where $u_{it}\,(i = 1,2,3)$ are the random errors which are assumed to the white noise generally, $x_t = x(t), y_t = y(t)$ *and* $z_t = z(t)$ indicate that the variables x, y *and* z depending on time t.

$$
\begin{aligned}
f_1(x_t, y_t, z_t, A_1) &= c_1 + a_{11} x_t + a_{12} y_t + a_{13} z_t + a_{14} x_t^2 + a_{15} y_t^2 + a_{16} z_t^2 + a_{17} x_t y_t + a_{18} y_t z_t + a_{19} z_t x_t, \\
f_2(x_t, y_t, z_t, A_2) &= c_2 + a_{21} x_t + a_{22} y_t + a_{23} z_t + a_{24} x_t^2 + a_{25} y_t^2 + a_{26} z_t^2 + a_{27} x_t y_t + a_{28} y_t z_t + a_{29} z_t x_t, \qquad (9) \\
f_3(x_t, y_t, z_t, A_3) &= c_3 + a_{31} x_t + a_{32} y_t + a_{33} z_t + a_{34} x_t^2 + a_{35} y_t^2 + a_{36} z_t^2 + a_{37} x_t y_t + a_{38} y_t z_t + a_{39} z_t x_t
\end{aligned}
$$

Where $A_i = (c_i, a_{i1}, a_{i2}, \ldots\ldots\ldots\ldots, a_{i9}), i = 1,2,3;$

Then the model can be rewritten as:

$$
\begin{aligned}
D_t^{q_1} x_t &= f_1(x_t, y_t, z_t, A_1) + u_{1t}, \\
D_t^{q_2} y_t &= f_2(x_t, y_t, z_t, A_2) + u_{2t}, \qquad (10) \\
D_t^{q_3} z_t &= f_3(x_t, y_t, z_t, A_3) + u_{3t}
\end{aligned}
$$

According to (4), when $0 < \alpha < 1$, the model (10) can be discretized as:

$$
\begin{aligned}
D_t^{q_1} x_t &= \frac{x(t_{n+1}) - x(t_n)}{(t_{n+1} - t_n)^{q_1}} \Gamma(1 + q_1) = f_1(x_t, y_t, z_t, A_1) + u_{1t}, \\
D_t^{q_2} y_t &= \frac{y(t_{n+1}) - y(t_n)}{(t_{n+1} - t_n)^{q_2}} \Gamma(1 + q_2) = f_2(x_t, y_t, z_t, A_2) + u_{2t} \qquad (11) \\
D_t^{q_3} z_t &= \frac{z(t_{n+1}) - z(t_n)}{(t_{n+1} - t_n)^{q_3}} \Gamma(1 + q_3) = f_3(x_t, y_t, z_t, A_3) + u_{3t}
\end{aligned}
$$

We estimate (11) based on empirical data to determine the relationship of these variables. From the form of the model (11), it is easy to find that there does not exists common parameters in three equations of it. Therefore, the above three multivariate regression equations can be estimated separately. To state the technical procedures, we take the first equation as an example. The estimation for the parameters in the other two equations is similar.

Consider $q_1 = 1$; then

$$\frac{x(t_{n+1})-x(t_n)}{(t_{n+1}-t_n)} = f_1(x_t, y_t, z_t, A_0) + u_{1t} \qquad (12)$$

Let $Y_i = \frac{x(t_{i+1})-x(t_i)}{(t_{i+1}-t_i)}$, $i = 1,2,\ldots\ldots\ldots\ldots N-1$; then define the least squares (LS) function as:

$$SSR\,(A_0) = \sum_{i=1}^{N-1}(Y_i - f_1(x(t_i), y(t_i), z(t_i), A_0)^2 \qquad (13)$$

The LS estimator of the regression parameter $\hat{A}_0 = \arg\min \sum_{i=1}^{N-1} (Y_i - f_1(x(t_i), y(t_i), z(t_i), A_0)^2$ (14)

For simplicity, we denote $X_{1i} = x(t_i), X_{2i} = y(t_i), X_{3i} = z(t_i), X_{4i} = x(t_i)^2, X_{5i} = y(t_i)^2, X_{6i} = z(t_i)^2, X_{7i} = x(t_i)y(t_i), X_{8i} = y(t_i)z(t_i), X_{9i} = z(t_i)x(t_i), i = 1,2, \ldots\ldots, N-1$, where N is the number of sample studied. Similar to the procedure of estimating the multivariate regression by the method of least squares, we can obtain the least squares estimator of the model as:

$$\hat{A}_0 = (X^T X)^{-1}(X^T Y), \quad (15)$$

where

$$X = \begin{pmatrix} 1 & X_{11} & X_{12} & \cdots & X_{19} \\ 1 & X_{21} & X_{22} & \cdots & X_{29} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & X_{(N-1)1} & X_{(N-1)2} & \cdots & X_{(N-1)N} \end{pmatrix} \quad (16)$$

and $Y = (Y_1, Y_2, \ldots\ldots, Y_{N-1})^T$. The subscript T indicates the transportation of matrix and vector.

We consider the first regression equation and estimate the parameters (q_1, A_1). The corresponding least squares estimation is subjected to:

$$(\hat{q}_1, \hat{A}_1) = \arg\min SSR(q_1, A_1) = \arg\min \sum_{n=1}^{N-1} \left(\frac{x(t_{n+1}) - x(t_n)}{(t_{n+1}-t_n)^{q_1}} \Gamma(1+q_1) - f_1(x(t_n), y(t_n), z(t_n), A_1)\right)^2 =$$
$$\arg\min \left(\Gamma(1+q_1)(t_{n+1}-t_n)^{1-q_1}\right)^2 \times \sum_{n=1}^{N-1} \left(\frac{x(t_{n+1}) - x(t_n)}{(t_{n+1}-t_n)} - f_1(x(t_n), y(t_n), z(t_n), A'_1)\right)^2, \quad (17)$$

where $A'_1 = \frac{(t_{n+1}-t_n)^{q_1-1} A_1}{\Gamma(1+q_1)}$ (18)

According to the evaluation result with $q_1 = 1$, the minimum of the second product part implies that $A'_1 = \hat{A}_0$. Hence,

$$\hat{q}_1 = \arg\min \{\Gamma(q_1+1)(t_{n+1}-t_n)^{1-q_1}\}, A'_1 = \hat{A}_0, \quad (19)$$

and the minimum of $SSR\ (q_1, A_1)$ can be obtained as:

$$\hat{A}_1 = \Gamma(\hat{q}_1 + 1)(t_{n+1}-t_n)^{1-\hat{q}_1}(X^T X)^{-1}(X^T Y), \quad (20)$$

$$\hat{q}_1 = \arg\min \{\Gamma(q_1+1)(t_{n+1}-t_n)^{1-q_1}\} \quad (21)$$

Equations (20)-(21) are the least squares estimation of q_1 and A_1 in the first regression equation of system (11), respectively. It is easy to find that the estimator of $q_1$is not related to the sample observations, and can be computed by numerically. Using the same technique, we can deal with q_2 and q_3 in system (11) and obtain the optimal estimators of $\hat{q}_2$ and $\hat{q}_3$. In the next section, the dynamics of the new model and prediction results based on the macroeconomic data of India is considered.

EMPIRICAL RESULTS OF DISCRETE FINANCIAL SYSTEM: EVIDENCE FROM INDIA

In this section, we present the study of discrete financial system based on the macroeconomic data of India.

Data Description

In financial model (11), the nonlinear dynamic behaviours of interest rate, investment demand and inflation are studied. This work chooses six-month London interbank offered rate (LIBOR) data to reflect interest rate change in India. The total investment percent of GDP is used to measure the investment demand. Average consumer prices percent change rate will be used to reflect the inflation. The annual data starts from year 1981to 2015. The data about LIBOR, investment percent of GDP, and average consumer prices percent rate are obtained from EconStats which is organized by IMF.

Empirical Results

The optimal fractional orders $q_i = 1{,}2{,}3$ with different step sizes $\Delta t = 1, 0.9, 0.8, 0.7{,}0.6$ are performed in Table 2. We do not consider the case of $\Delta t < 0.5$ because of the fact that the fractional order decreases and approaches to zero, which reduces this present model to be a linear one but not the fractional financial system.

$$\hat{Y}_i = 7.456 - 1.379x_t + 0.343y_t - 2.781z_t + 0.02x_t^2 + 0.005y_t^2 + 0.155z_t^2 + 0.066x_t y_t - 0.025y_t z_t + 0.064z_t x_t,$$

$$\hat{Y}_i = 7.456 - 1.379x_t + 0.343y_t - 2.781z_t + 0.02x_t^2 + 0.005y_t^2 + 0.155z_t^2 + 0.066x_t y_t - 0.025y_t z_t + 0.064z_t x_t,$$

$$\hat{Y}_i = 7.456 - 1.379x_t + 0.343y_t - 2.781z_t + 0.02x_t^2 + 0.005y_t^2 + 0.155z_t^2 + 0.066x_t y_t - 0.025y_t z_t + 0.064z_t x_t,$$

Table 2: Optimal fractional order q

Δt	1	0.9	0.8	0.7	0.6	0.5
Extended Solver Steps / Solver Iteration	2/18	2/17	2/18	2/14	2/16	2/12
$\hat{q}_i$	0.4616321	0.3579855	0.2536069	0.1482567	0.04157812	0
W	0.8856032	0.8321360	0.7667172	0.6890221	0.5991806	0.5

Note: When$\Delta t \leq 0.5$, $\hat{q}_i \leq 0$; thus the case of $\Delta t \leq 0.5$ is not to be considered and $W = \Gamma(\hat{q}_1 + 1)\Delta t 1 - q 1$.

Table 3: The first equation in () under the several time steps of dicretization

Parameters: Coefficients of the variables	Δt					
	1	0.9	0.8	0.7	0.6	1
	$\hat{q}_i$					
	0.4616321	0.3579855	0.2536069	0.1482567	0.04157812	1
c	6.603 (0.617)	6.204 (0.617)	5.716 (0.617)	5.137 (0.617)	4.467 (0.617)	7.456 (0.617)
x_t	-1.221 (0.183)	-1.147 (0.183)	-1.057 (0.183)	-0.950 (0.183)	-0.826 (0.183)	-1.379 (0.183)
y_t	0.303 (0.605)	0.285 (0.605)	0.263 (0.605)	0.236 (0.605)	0.205 (0.605)	0.343 (0.605)
z_t	-2.463 (0.472)	-2.314 (0.472)	-2.132(0.472)	-1.916 (0.472)	-1.666 (0.472)	-2.781 (0.472)
x^2	0.018 (0.043)	0.017 (0.043)	0.016 (0.043)	0.014 (0.043)	0.012 (0.043)	0.02 (0.043)
y^2	0.005 (0.867)	0.004 (0.867)	0.004 (0.867)	0.004 (0.867)	0.003 (0.867)	0.005 (0.867)
z^2	0.137 (0.511)	0.129 (0.511)	0.119 (0.511)	0.107 (0.511)	0.093 (0.511)	0.155 (0.511)
$xt\ yt$	0.058 (0.156)	0.055 (0.156)	0.051 (0.156)	0.045 (0.156)	0.039 (0.156)	0.066 (0.156)
$yt\ zt$	-0.022 (0.699)	-0.021 (0.699)	-0.019 (0.699)	-0.017 (0.699)	-0.015 (0.699)	-0.025 (0.699)
$zt\ xt$	0.056 (0.584)	0.053 (0.584)	0.049 (0.584)	0.044 (0.584)	0.038 (0.584)	0.064 (0.584)
R^2	0.681	0.681	0.681	0.681	0.681	0.681
SSR	1393.396	1230.226	1044.399	843.456	637.840	1776.627
$Prob(F)$	0.000	0.000	0.000	0.000	0.000	0.000

Note. Bracketed value denotes the statistical significance at the 5% levels. R^2 is the coefficient of determination, SSR is the sum of squared residuals, $Prob(F)$ is the P value of F- statistic.

Table 4: The first equation in () under the several time steps of dicretization

Parameters: Coefficients of the Variables	Δt					
	1	0.9	0.8	0.7	0.6	1
	$\hat{q}_i$					
	0.4616321	0.3579855	0.2536069	0.1482567	0.04157812	1
c	5.212 (0.632)	4.897 (0.632)	4.512 (0.632)	4.055 (0.632)	3.526 (0.632)	5.885 (0.632)
x_t	0.432 (0.561)	0.406 (0.561)	0.374 (0.561)	0.336 (0.561)	0.292 (0.561)	0.488 (0.561)
y_t	-0.689 (0.162)	-0.647 (0.162)	-0.596 (0.162)	-0.536 (0.162)	-0.466 (0.162)	-0.778 (0.162)
z_t	-0.778 (0.782)	-0.731 (0.782)	-0.674 (0.782)	-0.605 (0.782)	-0.526 (0.782)	-0.879 (0.782)
x_t^2	-0.005 (0.446)	-0.005 0.446)	-0.005 0.446)	-0.004 0.446)	-0.004 0.446)	-0.006 (0.446)
y_t^2	0.018 (0.437)	0.017 (0.437)	0.015 (0.437)	0.014 (0.437)	0.012 (0.437)	0.02 (0.437)
z_t^2	0.025 (0.886)	0.023 (0.886)	0.021 (0.886)	0.019 (0.886)	0.017 (0.886)	0.028 (0.886)
$x_t y_t$	-0.019 (0.562)	-0.018 (0.562)	-0.017 (0.562)	-0.015 (0.562)	-0.013 (0.562)	-0.022 (0.562)
$y_t z_t$	-0.046 (0.33)	-0.043 (0.33)	-0.040 (0.33)	-0.036 (0.33)	-0.031 (0.33)	-0.052 (0.33)
$z_t x_t$	-0.063 (0.461)	-0.059 (0.461)	-0.054 (0.461)	-0.049 (0.461)	-0.042 (0.461)	-0.071 (0.461)
R^2	0.618	0.618	0.618	0.618	0.618	0.618
SSR	944.441	833.845	707.892	571.693	432.327	1204.194
$Prob(F)$	0.000	0.000	0.000	0.000	0.000	0.000

Note. Bracketed value denotes the statistical significance at the 5% levels. R^2 is the coefficient of determination, SSR is the sum of squared residuals, $Prob(F)$ is the P value of F- statistic.

__Table 5: The first equation in () under the several time steps of dicretization__

Parameters: Coefficients of the Variables	Δt					
	1	0.9	0.8	0.7	0.6	1
	$\hat{q}_i$					
	0.4616321	0.3579855	0.2536069	0.1482567	0.04157812	1
c	-2.93 (0.46)	-2.753 (0.46)	-2.537 (0.46)	-2.28 (0.46)	-1.983 (0.46)	-3.309 (0.46)
x_t	-0.255 (0.349)	-0.239 (0.349)	-0.22 (0.349)	-0.198 (0.349)	-0.172 (0.349)	-0.287 (0.349)
y_t	-0.012 (0.945)	-0.011 (0.945)	-0.01 (0.945)	-0.009 (0.945)	-0.008 (0.945)	-0.014 (0.945)
z_t	1.413 (0.174)	1.328 (0.174)	1.223 (0.174)	1.099 (0.174)	0.956 (0.174)	1.595 (0.174)
x^2	0.003 (0.292)	0.003 (0.292)	0.002 (0.292)	0.002 (0.292)	0.002 (0.292)	0.003 (0.292)
y^2	-0.003 (0.671)	-0.003 (0.671)	-0.003 (0.671)	-0.003 (0.671)	-0.002 (0.671)	-0.004 (0.671)
z^2	-0.12 (0.063)	-0.113 (0.063)	-0.104 (0.063)	-0.093 (0.063)	-0.081 (0.063)	-0.135 (0.063)
$x_t y_t$	0.001 (0.967)	0.000 (0.967)	0.000 (0.967)	0.000 (0.967)	0.000 (0.967)	0.001 (0.967)
$y_t z_t$	0.008 (0.644)	0.007 (0.644)	0.007 (0.644)	0.006 (0.644)	0.005 (0.644)	0.009 (0.644)
$z_t x_t$	0.032 (0.311)	0.03 (0.311)	0.027 (0.311)	0.025 (0.311)	0.021 (0.311)	0.036 (0.311)
R^2	0.546	0.546	0.546	0.546	0.546	0.546
SSR	124.866	110.244	93.592	75.585	57.159	159.209
$Prob(F)$	0.011	0.011	0.011	0.011	0.011	0.011

Note. Bracketed value denotes the statistical significance at the 10% levels. R^2 is the coefficient of determination, SSR is the sum of squared residuals, $Prob(F)$ is the P value of F- statistic.

Table 3-5 show the results about the estimated coefficients, coefficient of determination, sum of residuals, and P statistical test values in empirical model equations for various discretization time intervals. Estimated coefficients and other values are also provided under the integer-order Chen system for comparison.

Table 3 indicates that the coefficient of determination (R2) for the nine independent variables is 0.681, indicating that this set of factors adequately explains 68.1% of the variance in interest rates. It's worth noting that F = 0.0001. This means there is a 99.999% chance that the model is correct. For each independent variable, we also have its t test result, which indicates its significance at the 95% level of confidence. The sole statistically significant variable in this table is x2, the dynamic independent variable, with a value of 0.043 (less than 0.05). The remaining eight dynamic independent variables do not add up to statistical significance on their own. In conclusion, the empirical equation for the interest rate comprises a structure of components whose corresponding coefficient estimates are significant at the 5% level.

x, y, z, x2, y2, z2, xy, yz, zx, constant term, and does not rely on the amount of the temporal discretization. Since qi estimate in (21) is independent of sample data, the result makes sense. Nonlinear financial models with a changing interest rate often have a larger sum of squares owing to residuals if the order of the model is integer rather than fractional. Therefore, the fractional nonlinear systems provide deeper insights into the dynamic behavior of the Indian financial sector.

Table 4 demonstrates that the nine independent factors account for 61.8% of the variance in investment, as measured by the R2 value of 0.618. It's worth noting that F = 0.0001. This means there is a 99.999% chance that the model is correct. In addition, we have the significance level of the test for each independent variable, expressed as a test value with a 95% degree of confidence. This table shows that none of the nine dynamic independent variables are statistically significant when considered separately. Finally, the structure of terms at which corresponding coefficients computed are significant at the 5% level in the empirical equation regarding investment rate contains terms x, y, z, x2, y2, z2, xy, yz, zx, constant term, and is insensitive to the time step size discretization. Since qi estimate in (21) is independent of sample data, the result makes sense.Integer-order nonlinear financial models of dynamic investments have a larger sum of squares owing to residual than their fractional-order counterparts. Therefore, the fractional nonlinear systems provide deeper insights into the dynamic behavior of the Indian financial sector.

Table 5 demonstrates that the nine independent variables account for 54.6% of the variance in inflation, as measured by the R2 value of 0.546. It's worth noting that F = 0.0001. This means there is a 99.999% chance that the model is correct. Additionally, we have the t test result for the significance of independent variables at the 90% confidence level. The only statistically significant value in this table is 0.063 for the dynamic independent variable z2, which is less than 0.1 (significant). The remaining eight dynamic independent variables do not add up to statistical significance on their own. Finally, the structure of terms at which corresponding coefficients estimated are significant at the 5% level in the inflation empirical equation includes terms x, y, z, x2, y2, z2, xy, yz, zx, constant term, and is insensitive to the time step size discretization. Since qi estimate in (21) is independent of sample data, the result makes sense.In the integer-order nonlinear financial model of dynamic inflation, the sum of squares owing to residual is larger than in the fractional-order models. Therefore, the fractional nonlinear systems provide deeper insights into the dynamic behavior of the Indian financial sector.

In Figures 1-3, we see the interest rate, investment, and inflation data as well as the data projected by the empirical model for t = 0.6. The black line in Figure 1 cuts through the bulk of the circle. We find that the model's empirical interest rate equation well describes both the estimated data (shown by the circles) and the actual data (represented by the black line). The black line in Figure 2 virtually touches all of the circles. Ideal empirical equation fitting of real-world investment data is represented by the property of the figure. Figure 3 shows a brief portion of time at the beginning of the era with a handful of circle notations off the black line. Based on the facts shown in Figure 3, it seems that the empirical equation about inflation is a good match. Figures 1-3 in particular provide strong evidence that the estimated empirical model follows the paper's stated approach and faithfully represents the underlying data.

Multistep interest rate, investment, and inflation predictions for (t = 1 and q = 1), (t = 1 and q = 0.9), (t = 1 and q = 0.6), and (t = 1 and q = 1) are displayed in Figures 4 and 5, respectively, to evaluate the efficacy of the conclusion regarding the prediction of the empirical model. Figure 4 shows that the predicted interest rate is fairly close to the actual interest rate. It implies that in all six scenarios of fractional order and integer first order, interest rate prediction is relevant. Figure 5 shows that the estimates are within a small margin of the actual expenditure. It indicates that in all six scenarios of fractional order and integer first order, investment forecast is significant. Figure 6 shows that the forecasts are quite consistent with the observed inflation. In all six scenarios, including integer first order, it implies that the inflation forecast is meaningful. Based on these findings, it seems that meaningful estimate of an empirical model of interest rates, investments, and inflation may be achieved using fractional-order optimization with a reasonable t.

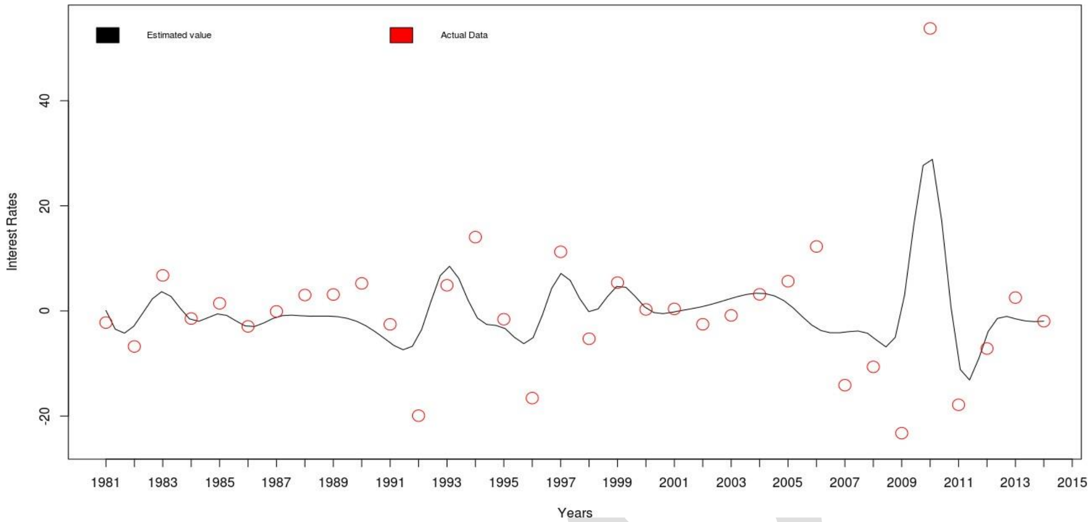

Figure 1: The actual interest rate versus estimated interest rate

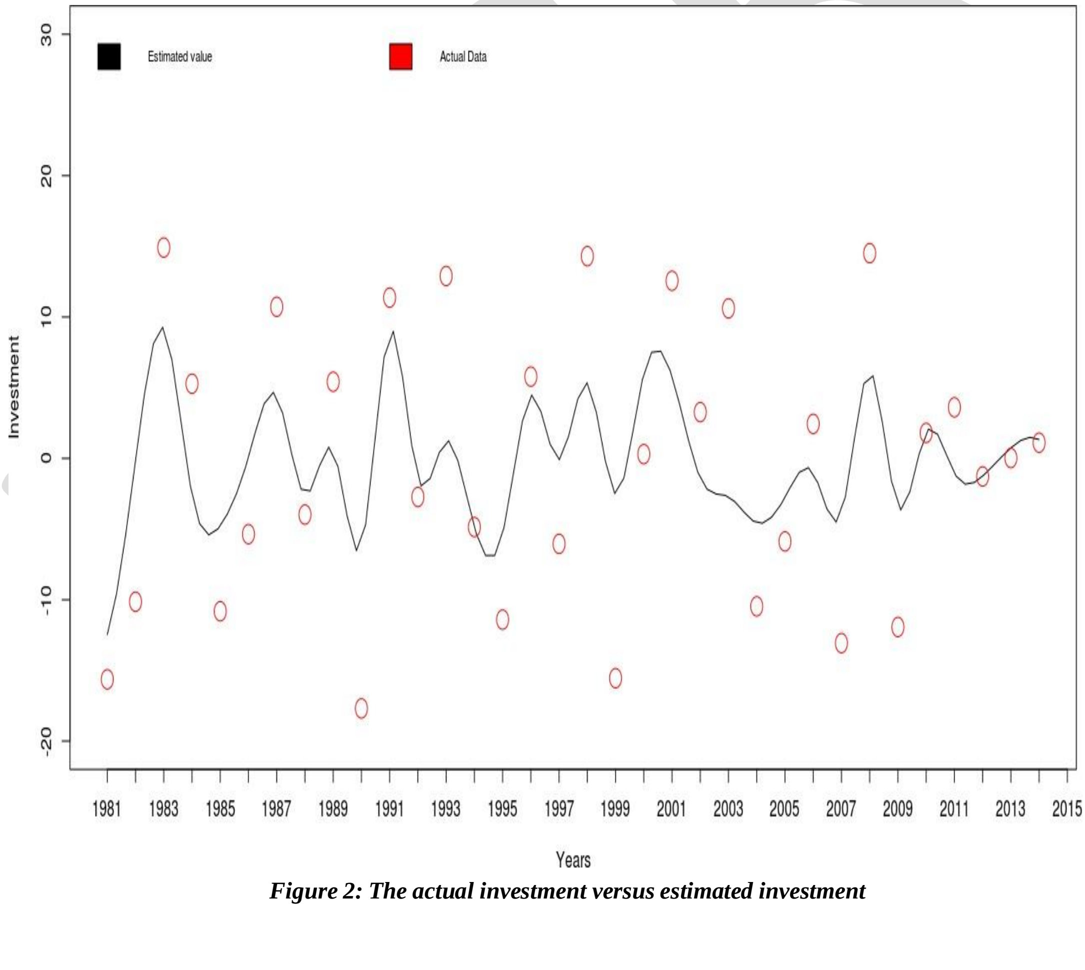

Figure 2: The actual investment versus estimated investment

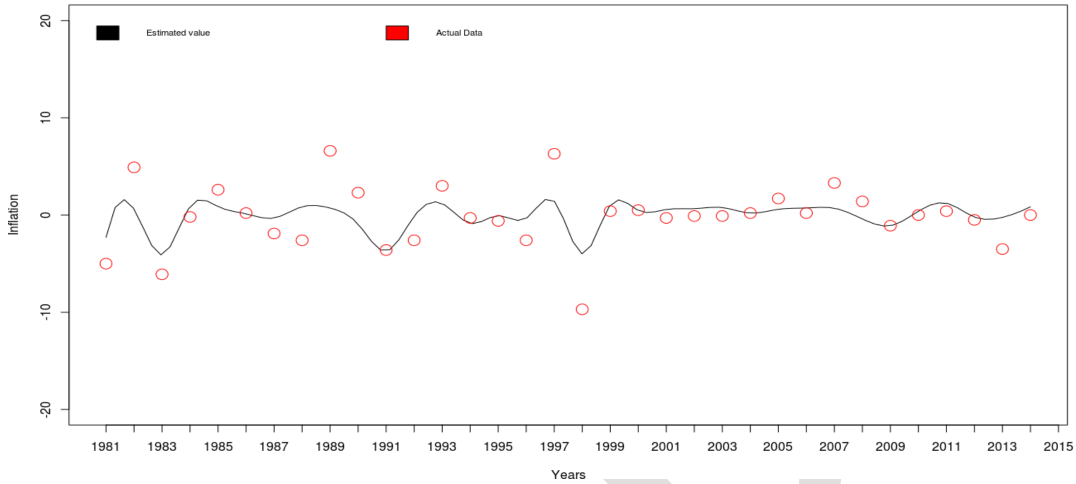

Figure 3: The actual inflation versus estimated inflation

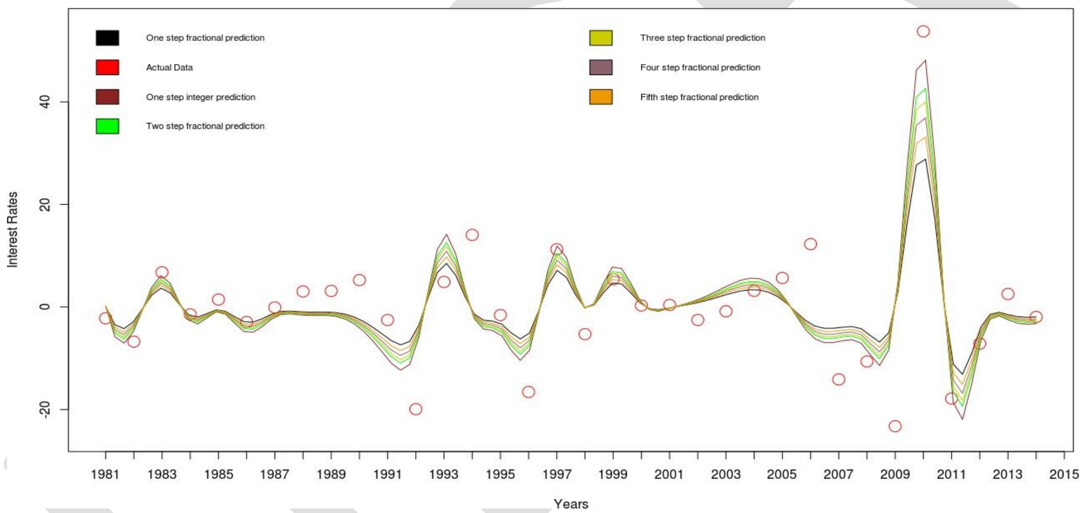

Figure 4: Fractional-order predictions of interest rate on empirical model

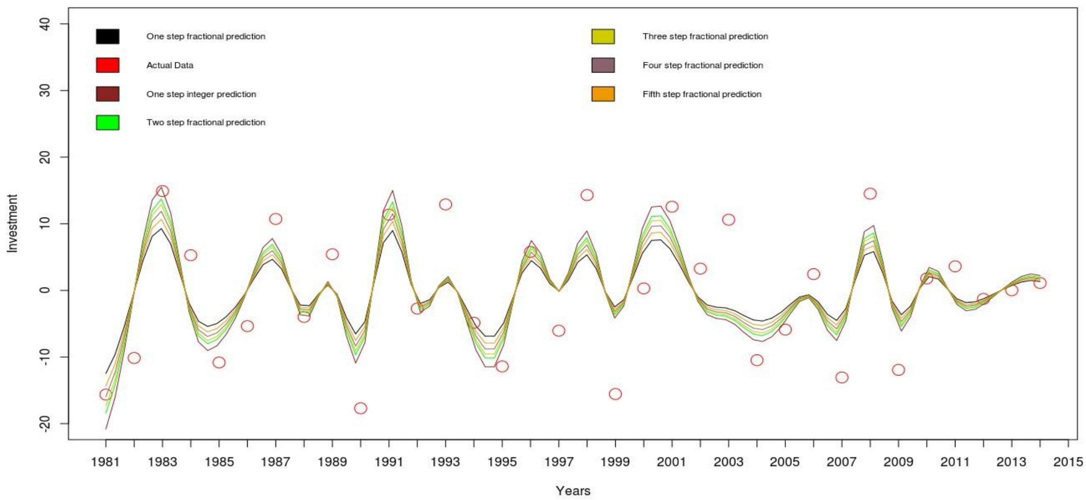

Figure 5: Fractional-order predictions of investment on empirical model

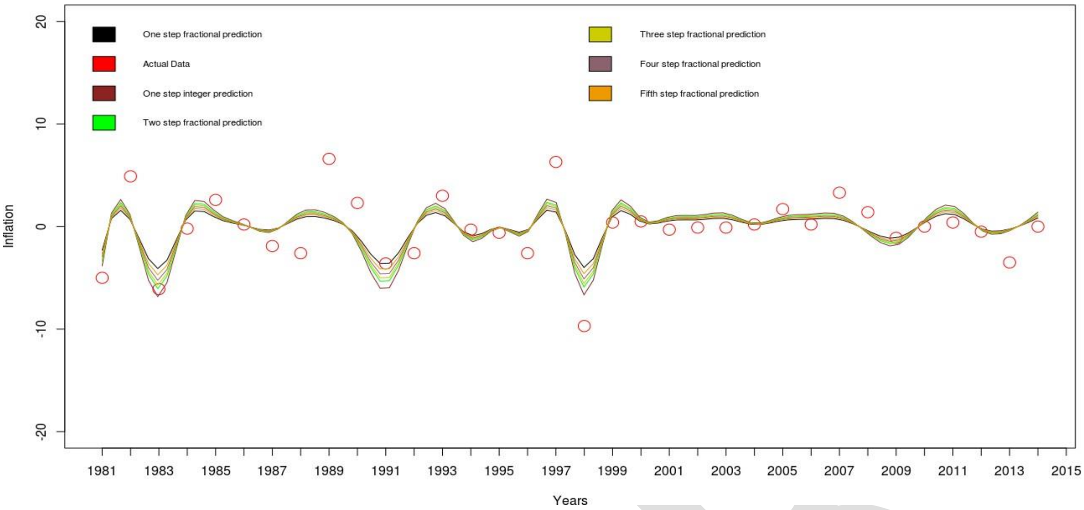

Figure 6: Fractional-order predictions of inflation on empirical model

CONCLUSIONS

Recent reports have suggested a fractional model of finance as an alternative to integer-order models. In this research, we use Jumarie's fractional-order derivative to present a novel nonlinear dynamic financial econometric model. In order to create the discrete financial model that corresponds to Jumarie's derivative, the limit operation must be removed. This model solves the issues that other nonlinear financial models in the literature cannot, and it gives a workable method for representing the real macroeconomic data of a single region using a nonlinear model. Empirical study confirms the current model's reasonableness.

We assess the model's settings based on information about India's economy. India's data is then converted to the best fractional order possible. Observations from the empirical research show that the fractional order does, in fact, affect the dynamic behavior of financial systems. This innovative fractional financial model may be used to anticipate and shed light on the dynamic behavior of India's financial systems in the years to come, thanks to its optimum fractional order.

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