EFFECT OF BINARY CHEMICAL REACTION ON UNSTEADY THREE-DIMENSIONAL MHD FLOW OF POWELL-EYRING NANO FLUID

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ABSTRACT

In the present investigation, the effect of binary chemical reaction on three dimensional MHD flow of Powell-Eyring nanofluid over stretching sheet is analyzed numerically. Mathematical model has been formulated to construct continuity, momentum, energy and concentration equations. The governing boundary layer equations are reduced into system of nonlinear ordinary differential equations by using similarity transformation and then solved by Runge-kutta method. The influence of non-dimensional parameters Powell-Eyring fluid parameter ε, unsteady parameter S, magnetic parameter M, Prandtl number Pr, Lewis number Le, Thermophoresis number Nt, Brownian number Nb, non-dimensional energy E, Biot number Bi, temperature difference parameter δ, dimensionless reaction rate σ and fitted rate constant n respectively on velocity, temperature and concentration profiles are shown numerically and graphically with the help of graphs.

Keywords: MHD, Powell Eyring-fluid, stretching sheet, Binary chemical reaction, Activation energy.

1. INTRODUCTION


2. MATHEMATICAL FORMULATION

Let us consider three-dimensional boundary layer flow of an electrically conducting Eyring-Powell fluid past a convectively heated stretching sheet. Let \( x, y, z \) be the Cartesian coordinates axis with the origin \( O \) and \( u, v, w \)
represent the velocity components in the directions x, y, and z respectively. At \( z=0 \), the sheet coincides with the plane and the flow occupies the region when \( z \) is greater than 0. By fixing the origin as O, the sheet is stretched in two directions x and y with the velocities \( u \) and \( v \) in the form:

\[
\begin{align*}
\frac{\partial u}{\partial x} &= \frac{ax}{1 - \alpha t} \\
\frac{\partial v}{\partial y} &= \frac{by}{1 - \alpha t}
\end{align*}
\] (1)

where \( a \) and \( b \) are positive constants.

The convective temperature and concentration of nanoparticles at the sheet are \( T_f \) and \( C_w \) and \( T_\infty \) and \( C_\infty \) denote the ambient fluid temperature and concentration. The transverse magnetic field is taken in the following form:

\[
B = \frac{B_0}{(1 - \alpha t)^{1/2}}
\] (2)

The Cauchy stress tensor \( T \) for Powell-Eyring fluid model is as follows

\[
T = -pI + \tau
\] (3)

\[
\rho_f \alpha_i = -\nabla p + \nabla \cdot (\tau_{ij}) + \sigma J \times B
\] (4)

In this Powell-Eyring fluid model, \( p \) is represent pressure, \( I \) is identity tensor and \( \tau_{ij} \) extra stress tensor

\[
\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta} \sinh^{-1}\left(\frac{1}{\gamma} \frac{\partial u_i}{\partial x_j}\right)
\] (5)

where \( \beta \) and \( \gamma \) are the characteristic length.

Let us considering

\[
\sinh^{-1}\left(\frac{1}{\gamma} \frac{\partial u_i}{\partial x_j}\right) \approx \frac{1}{\gamma} \frac{\partial u_i}{\partial x_j} - \frac{1}{6} \left(\frac{1}{\gamma} \frac{\partial u_i}{\partial x_j}\right)^3 \frac{1}{\gamma} \frac{\partial u_i}{\partial x_j} < 1
\] (6)

The governing three-dimensional Eyring-Powell fluid equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\] (7)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \nu \frac{\partial^2 u}{\partial x^2} + \frac{\mu}{\beta} \frac{\partial^2 u}{\partial y^2} - \frac{1}{2} \frac{2\beta^3}{\gamma^3} \frac{\rho_f}{\rho_f} \left( \frac{\partial u}{\partial z} \right)^2 \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B^2}{\rho_f} u
\] (8)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + \nu \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = \nu \frac{\partial^2 v}{\partial x^2} + \frac{\mu}{\beta} \frac{\partial^2 v}{\partial y^2} - \frac{1}{2} \frac{2\beta^3}{\gamma^3} \frac{\rho_f}{\rho_f} \left( \frac{\partial v}{\partial z} \right)^2 \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B^2}{\rho_f} v
\] (9)

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} = \alpha_m \frac{\partial^2 T}{\partial z^2} + \tau \left\{ D_T \left( \frac{\partial C}{\partial z} \right) + D_r \left( \frac{\partial T}{\partial z} \right)^2 \right\}
\] (10)

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} + \frac{\partial C}{\partial z} = D_T \left( \frac{\partial^2 C}{\partial z^2} \right) + D_r \left( \frac{\partial T}{\partial z} \right)^2 - K^2 (C - C_\infty) \left( \frac{T}{T_\infty} \right) \left( \frac{E_k}{s^2} \right)
\] (11)
The above velocity components \( u, v \) and \( w \) along with \( x, y \) and \( z \) directions, \( \nu \) kinematic viscosity, \( \mu \) dynamic viscosity, \( \sigma \) electrical conductivity, \( \alpha_n = \frac{k}{\rho_f c_f} \) thermal diffusivity of the fluid, \( T \) and \( C \) are temperature and volume fraction of nanoparticles, \( k \) thermal conductivity of the fluid, \( \tau = \frac{(\rho c)_h}{(\rho c)_f} \) ratio of the effective heat capacity of nanoparticle and \( D_B \) Brownian diffusion coefficient, \( D_T \) thermophoretic diffusion coefficient. \( K^2(C-C_e) \left( \frac{T}{T_c} \right)^n \exp \left( \frac{E_a}{RT} \right) \) is the modified Arrhenius equation: \( K^2 = \) reaction rate, \( E_a = \) activation energy, \( K = 8.61 \times 10^5 \text{eV/K} \) the Boltzmann constant and \( n = \) fitted rate constant and \( n \) lies between -1 and 1.

The boundary conditions are:

\[
\begin{align*}
\frac{\partial u}{\partial z} &= 0, \quad \frac{\partial v}{\partial z} = 0, \quad \frac{\partial w}{\partial z} = h_f (T - T_f), \quad C = C_w \text{ at } z = 0 \\
u &\rightarrow 0, v \rightarrow 0, \frac{\partial u}{\partial z} \rightarrow 0, \frac{\partial v}{\partial z} \rightarrow 0, T \rightarrow T_c, C \rightarrow C_c \text{ as } z \rightarrow \infty
\end{align*}
\]

(12)

Similarity transformations are

\[
\begin{align*}
\eta &= \frac{ax}{(1-\alpha t)^{1/2}}, \quad \gamma = \frac{by}{(1-\alpha t)^{1/2}}, \quad \Theta = \frac{T - T_c}{T_f - T_c}, \quad \Phi = \frac{C - C_e}{C_w - C_c}, \quad \eta = \sqrt{\frac{a}{v_f(1-\alpha t)}} z
\end{align*}
\]

(13)

The governing Powell-Eyring equations are reduced into the set of non-linear ordinary differential equations as follows

\[
\begin{align*}
(1 + \varepsilon) f'' + (f + g) f' - (f')^2 - S(f + \frac{1}{2} \eta f') - \varepsilon \delta_1 (f')^2 f' - M^2 f' &= 0 \\
(1 + \varepsilon) g'' + (f + g) g' - (g')^2 - S(g + \frac{1}{2} \eta g') - \varepsilon \delta_2 (g')^2 g' - M^2 g' &= 0 \\
\Theta'' + \text{Pr}(f + g) \Theta + \text{Pr} \text{Nb} \Phi \Theta + \text{Pr} \text{Nt} \Theta^2 &= 0 \\
\Phi'' + \text{Le}(f + g) \Phi' + \frac{\text{Nt}}{\text{Nb}} \Theta'' = \text{Le} \sigma(1 + \delta \Theta)^a \Phi \left( \frac{E}{1 + E^2} \right) &= 0
\end{align*}
\]

(14), (15), (16), (17)

The reduced boundary conditions become

\[
\begin{align*}
f &= 0, \quad g = 0, \quad f' = 1, \quad g' = c, \quad \Theta = B i(\eta - 1), \quad \Phi = 1 \text{ at } \eta = 0 \\
f' &\rightarrow 0, g' \rightarrow 0, f' \rightarrow 0, g' \rightarrow 0, \Phi \rightarrow 0, \Theta \rightarrow 0 \text{ as } \eta \rightarrow \infty
\end{align*}
\]

(18)

Where \( \delta_1, \delta_2 \) and \( \varepsilon \) are Eyring Powell parameter, \( c \) is stretching ratio parameter, \( M^2 \) is magnetic parameter, \( S \) is unsteady parameter, \( \text{Pr} \) is Prandtl number, \( \text{Nt} \) is Brownian motion parameter, \( \text{Nt} \) is thermophoresis
parameter, $R$ is thermal radiation parameter, $\text{Ec}_x$ is Eckert number along x direction, $\text{Ec}_y$ is Eckert number along y direction, $Bi$ is Biot’s number and $Le$ is Lewis number.

The non-dimensional parameters which are involved in this problem are

$$\delta_1 = \frac{u_w^2}{2 \nu C_x^2}, \delta_2 = \frac{v^3}{2 \nu C_y^2}, E = \frac{1}{\mu \beta C}, c = \frac{b}{a}, Le = \frac{v}{D_B}, M^2 = \frac{\sigma B_0^2}{\rho_f a}, \sigma = \frac{K^2}{c}, Pr = \frac{v}{\alpha_m},$$

$$\delta = \frac{T_w - T^\infty}{T^\infty}, Nb = \frac{\tau D_f (C_w - C_{w_0})}{\nu}, Bi = \frac{h_f}{k \sqrt{\nu/a}}, Nt = \frac{\tau D_f (T_f - T^\infty)}{T_v^\nu}, E = \frac{E_a}{\kappa T}, S = \alpha \frac{a}{a} \quad (19)$$

3. RESULTS AND DISCUSSION

In this section, the effect of binary chemical reaction on three dimensional MHD flow of Powell-Eyring nanofluid over stretching sheet is analyzed numerically. The governing boundary layer equations are reduced into system of nonlinear ordinary differential equations by using similarity transformation and then solved by Runge-kutta method. The influence of non-dimensional parameters Powell-Eyring fluid parameter $\varepsilon$, unsteady parameter $S$, magnetic parameter $M$, Prandtl number $Pr$, Lewis number $Le$, Thermophoresis number $Nt$, Brownian number $Nb$, non-dimensional energy $E$, Biot number $Bi$, temperature difference parameter $\delta$, dimensionless reaction rate $\sigma$ and fitted rate constant $n$ respectively on velocity, temperature and concentration profiles are shown numerically and graphically with the help of graphs.

Figure 1 explores the various values of Powell-Eyring fluid parameter $\varepsilon$ on velocity profiles $f'$ and $g'$. Increasing values of Powell-Eyring fluid parameter $\varepsilon$ increase both profiles. Higher values of Powell-Eyring fluid parameter $\varepsilon$ reduced the boundary layer thickness. Figure 2 depicts unsteady parameter $S$ on velocity profiles $f'$ and $g'$ which reduces $f'$ and $g'$ initially. This is because of decreasing values in stretching rate. Influence of magnetic parameter $M$ on velocity profiles $f'$ and $g'$ are presented in the figure 3 and it was observed that increasing values of magnetic parameter $M$ decrease the profiles $f'$ and $g'$. Figure 4 illustrates the behaviour of unsteady parameter $S$ on temperature profile and concentration profile respectively. Enhancing values of unsteady parameter $S$ increase temperature profile and concentration profile. In figure 5, comparative numerical analysis of non-dimensional parameter Prandtl number $Pr$ on temperature profile and Lewis number $Le$ on concentration profile are shown. Increasing values of Prandtl number $Pr$ decreases temperature profile which diminishes the boundary layer thickness. With increasing values of Lewis number $Le$, concentration profile decreases. Figure 6 and 7 demonstrates the effect of Thermophoresis number $Nt$, Brownian number $Nb$ on temperature profile and concentration profile respectively. It is noticed that increasing values of Thermophoresis number $Nt$ increase both profiles but increase in Brownian number $Nb$ increases temperature profile and decrease the concentration profile. Increasing values of Biot number $Bi$ increase temperature profile and concentration profile respectively in figure 8. The effect of non-dimensional parameters non-dimensional energy $E$, temperature difference parameter $\delta$, dimensionless reaction rate $\sigma$ and fitted rate constant $n$ related to binary chemical reaction on concentration profile are revealed in the figure 9-10. Concentration profile increases with larger values of temperature difference parameter $\delta$, non-dimensional energy $E$ and dimensionless reaction rate $\sigma$ and it can be observed that opposite reaction for fitted rate constant $n$. As a result, it can be observed that chemical reaction requires more activation energy when mass flux of stretching sheet is smaller.
4. CONCLUSION

Numerical solutions of three dimensional MHD flow of Powell-Eyring nanofluid over stretching sheet with binary chemical reaction are presented. The main outcome of the present study are as follows:

- Increasing values of Powell-Eyring fluid parameter $\varepsilon$ increase both profiles. Higher values of Powell-Eyring fluid parameter $\varepsilon$ reduced the boundary layer thickness.
- Unsteady parameter $S$ on velocity profiles $f'$ and $g'$ which reduces $f'$ and $g'$ initially. This is because of decreasing values in stretching rate.
- Increasing values of magnetic parameter $M$ decrease the profiles $f'$ and $g'$.
- Enhancing values of unsteady parameter $S$ increase temperature profile and concentration profile.
- Increasing values of Prandtl number $Pr$ decreases temperature profile which diminishes the boundary layer thickness.
- With increasing values of Lewis number $Le$, Concentration profile decreases.
- It is noticed that increasing values of Thermophoresis number $Nt$ increase both profiles but increase in Brownian number $Nb$ increases temperature profile and decrease the concentration profile.
- Increasing values of Biot number $Bi$ increase temperature profile and concentration profile respectively.
- Concentration profile increases with larger values of temperature difference parameter $\delta$, non-dimensional energy $E$ and dimensionless reaction rate $\sigma$ and it can be observed that opposite reaction for fitted rate constant $n$.
- It can be observed that chemical reaction requires more activation energy when mass flux of stretching sheet is smaller.

REFERENCES


